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PRACTICAL GEOMETRY:

Or, A New and Easy

METHOD

OF

Treating that ART.

QA
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1727

WHEREBY

The Practice of it is render'd plain and familiar, and the Student is directed in the moſt eaſy manner thro' the ſeveral Parts and Progreſſions of it.

Tranſlated from the French of
Monſieur *S. LE CLERC.*

THE THIRD EDITION.

Illuſtrated with eighty Copper-Plates.

Wherein, beſides the ſeveral Geometrical Figures, are contain'd many Examples of Landskips, Pieces of Architecture, Perspective, Draughts of Figures, Ruins, &c.

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OF GEOMETRY IN GENERAL.

GEometry is a Greek word, and in its native ſignification ſtands for no more than the meaſuring of land, but now we mean by it the principal part of Mathematicks, which is a ſcience that has continued quantity for its object.

That quantity is call'd continued quantity, which has all its parts conjoin'd; of this kind are all ſorts of extenſion, magnitudes, and dimenſions.

And theſe dimenſions conſiſt chiefly either in lines, or ſurfaces, or angles, or bodies, which laſt are not to be conſider'd in reſpect of the quality of their matter, but of the extenſion of their parts.

Geometry is divided into ſpeculative and practical.

The former is a ſcience that teaches the mind how to form ideas of, and demonſtrate the truth of geometrical propoſitions.

The latter, or practical Geometry, conducts the hand in working.

THE
Original of Geometry.

GEometry had its original among the *Egyptians*, who were put under a neceſſity of inventing ſome ſuch art to remedy the diſorders, that commonly happen'd in their lands, by the overflowing of the *Nile*, which carried away their land-marks, and effaced the limits of their inheritances.

So that this practice, which in thoſe days conſiſted only in meaſuring of land, that every one might have what belonged to him before the overflow, was call'd Geometry.

But in proceſs of time, the *Egyptians* applied themſelves to more ſubtle inquiries, and by degrees inſenſibly there aroſe from a practice altogether mechanical, a ſcience that now holds the firſt place among all the others, according to its merit.

THE

Uſefulneſs of Geometry.

GEometry is not only uſeful, but in ſeveral caſes neceſſary. 'Tis owing to this, that Aſtronomers are put into a way of making their obſervations, coming at the knowledge of the extent of the heavens, duration of time, motions of the heavenly bodies, meaſures of ſeaſons, of years, and of ages.

'Tis by the aſſiſtance of this ſcience that Geographers preſent to our view at once, the magnitude of the whole earth, the vaſt extent of the ſeas, the diviſions of Empires, Kingdoms, and Provinces.

'Tis from this that the Architects take their juſt meaſures for the ſtructure of publick buildings, as well as of private houſes.

By its help Engineers conduct all their works, take the ſituation and plan of places, meaſure their diſtances from one another, and carry their meaſure into places that are only acceſſible by the eye.

Perſons of Quality, whoſe birth engages them to take the field, are oblig'd to apply themſelves to this ſcience. It not only ſerves as an introduction into the art of Fortification, which teaches how to raiſe proper baſtions for the defence of places, and to raiſe and manage machines, that may ſerve to overturn or make breaches in thoſe of the enemy; but alſo brings them to great skill and dexterity in the art of war, in forming an army into order for battle, in encamping, dividing the ground for quartering the army, taking maps of counties, plans of towns, forts, and caſtles, meaſuring all ſorts of dimenſions, both acceſſible and inacceſſible, and in forming deſigns; finally to recommend them as much for their skill and addreſs, as for their ſtrength and courage.

All ſuch as profeſs the Art of Deſigning, ought to know ſomething of Geometry, ſeeing that without it they can't make themſelves maſters of Architecture, nor Perſpective, which are two parts abſolutely neceſſary to their art.

15

THE

PRINCIPLES

OF

GEOMETRY.

THE

Principles of Geometry.

GEometry is built upon three ſorts of principles, *viz.* Definitions, Axioms, and Petitions.

Definitions are brief explications of the names of things, or terms of art.

Axioms are propoſitions ſo true and evident, that 'tis impoſſible to queſtion or conteſt their truth.

Petitions are demands ſo eaſy and intelligible, that the execution and putting them in practice, require no demonſtration.

THE

DEFINITIONS.

THE

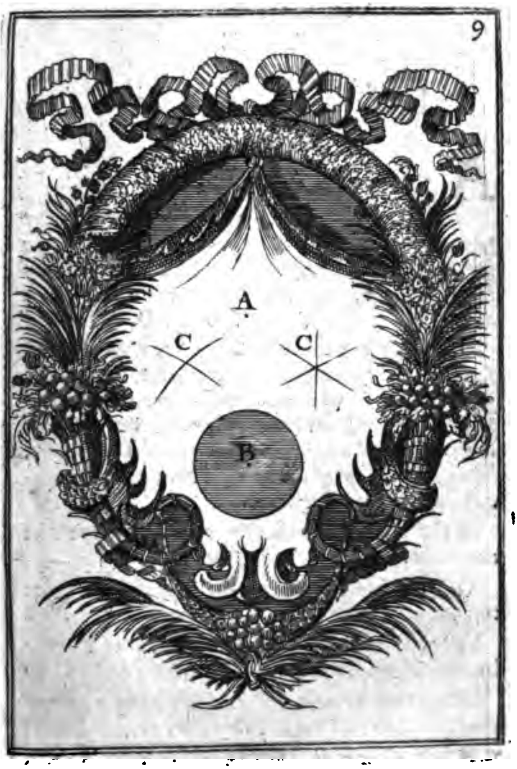
Definition of a Point.

A Point is that which has no parts.

By this definition you may eaſily perceive, that a point has neither length, nor breadth, nor depth; that it is not any thing ſenſible, but only intellectual; for nothing falls under the notice of our ſenſes, that has nothing of quantity, and nothing is quantity, that has not parts; ſo that to ſay a point is ſenſible, would be to ſay it has parts, which would contradict this definition. Notwithſtanding ſince no operation can be perform'd without the intervention of ſomething corporeal, we uſually repreſent a mathematical point by a phyſical point, which is an object of ſight the ſmalleſt and the leaſt ſenſible that can be, and which has no geometrical magnitude diviſible to our ſenſes, and is made by the prick of a pin, point of a compaſs, pen or pencil, as the point marked *A*

A central point, or centre is a point from which a circle, or a circumference is deſcribed; or rather it is the middle of a figure, as the point B

A ſecant point, or as ſome call it, a point of interſection, is a point where two or more lines croſs one another, as the point C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THE

Definition of a Line.

A Line is a length without any breadth.

A line is nothing but the track made by a point paſſing from one place to another, and would not be perceived, if it were not delineated by a phyſical point, which by its motion repreſents a line to us, as *A B, C D, E F*

There are as many ſorts of lines as there are different kinds of motions, which a point, the principle of a line, is capable of; tho' there are but two which are ſimple and the principal, viz. *a right and curve, and a third which is call'd a mixt line, becauſe made up of the two former, that are uſually conſider'd in Geometry.*

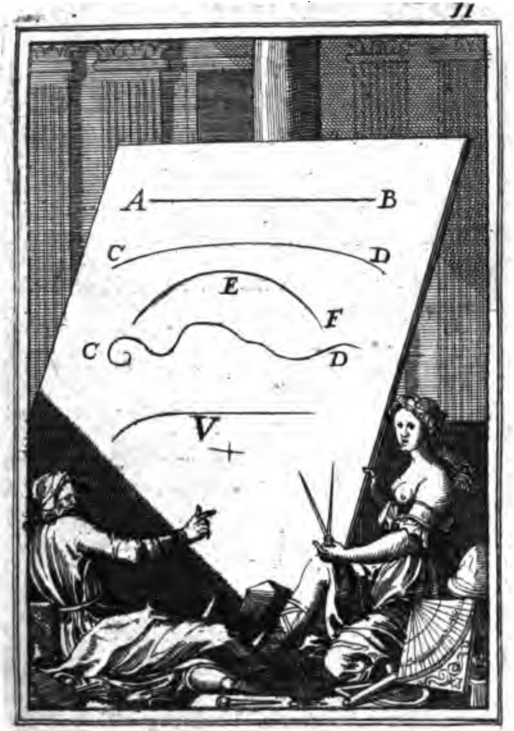
A right line is one that lies equally between its extremities.

Otherwiſe, 'tis a line that goes from one point to another without any deviation, as A B

A curve line is that which turns out of its way by one or more deviations, as C D

When ſuch a line as this is deſcribed by a pair of compaſſes, 'tis call'd a circular line, as E

A mixt line is that which is both ſtrait and a curve, as the line V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V

A line is diſtinguiſh'd into finite and infinite, into apparent and occult.

A Finite line is a bounded line, containing or ſuppoſing a neceſſary length, as A

An infinite line is an undetermin'd line, having no preciſe length, as B

An apparent line is one deſcribed with ink or a pencil, as A B

An occult, or white line, is only made with the point of a pair of compaſſes, or mark'd by points, and then 'tis call'd a prick'd line, as C

13

A line receives alſo ſeveral denominations, according to its different poſitions and properties.

A Perpendicular is a right line that is let fall or erected upon another, making the angles on each ſide equal, as A B

A Plumb line is that which hangs down without inclining to the right or to the left, and would paſs thro' the center of the earth, if it were produced infinitely, as C

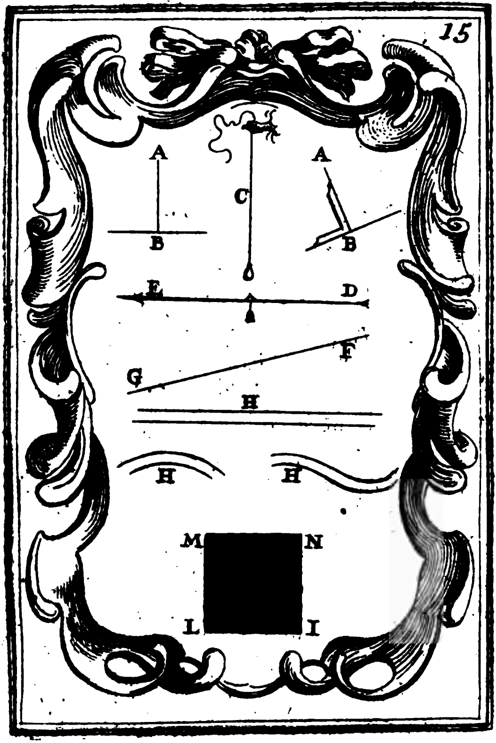
An Horizontal line is a line in equilibrio, equally inclin'd on both ſides. D E

Parallel lines are ſuch as follow one another at an equal diſtance H

An Oblique line is one that is neither horizontal nor perpendicular F G

A Baſe is a line upon which the figure reſts, as I L

Sides are the lines that contain a figure, as I N, L M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A Diagonal is a right line croſſing a figure, and terminated at its two oppoſite angles A B

A Diameter is a right line paſſing through the center of a circle, and terminated at the circumference C D

A Spiral line is a curve line iſſuing from a center, and continually going off from it at every turn E F

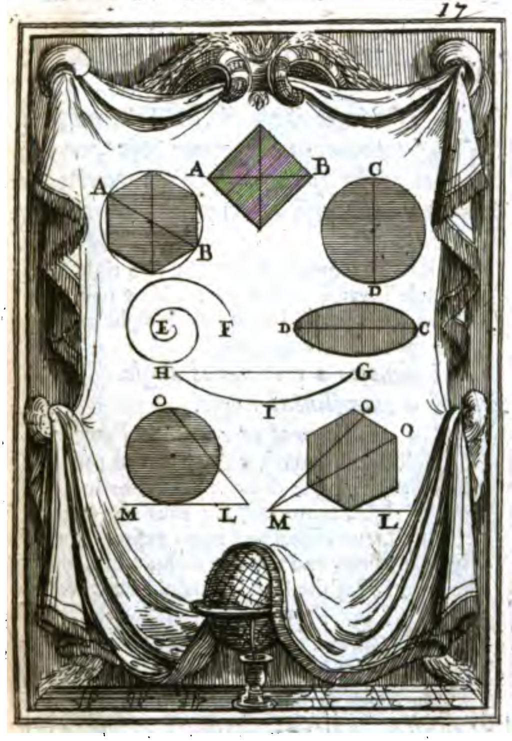
A Chord or ſubtenſe is a right line that joins the two extremities of an arc G H

An Arc is any part of a circumference G I H

A Tangent is a line that touches a figure without cutting it, nor would it cut or croſs the figure, though it were produced, as L M

A Secant is a line that does croſs or cut a figure L O, M O

If two lines meet at their extremities, they meet either directly or indirectly; if directly, they make but one line, if indirectly, they form an angle.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* * *

THE

Definition of an Angle.

AN Angle is the indirect concourſe of two lines in the ſame point, or rather it is the ſpace contain'd between the indirect concourſe of two lines meeting in a point, as A B C

If the concourſe be form'd by two right lines, the angle is call'd a Rectilineal, if by two curve lines, a Curvilineal, but if by one right and one curve line, a Mixtilineal angle.

A denotes a rectilineal angle.

B a curvilineal angle.

C a mixtilineal or compound angle.

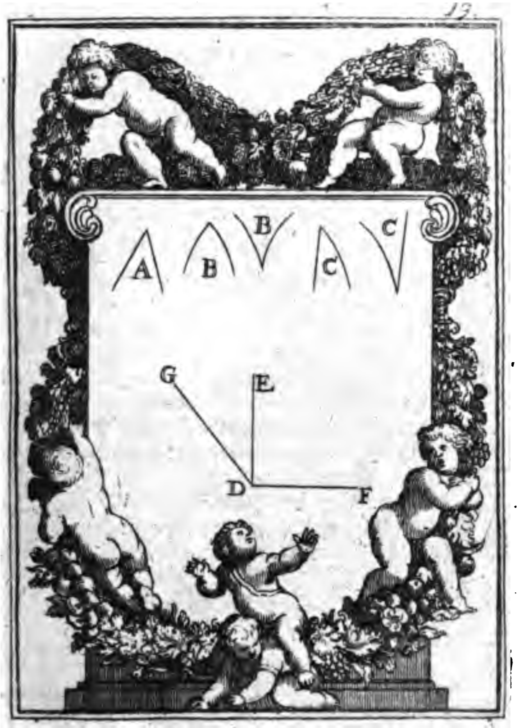
A rectilineal angle receives ſeveral particular names, according as it has a greater or leſs aperture, as right, acute, obtuſe; thus the terms of rectilineal, curvilineal and mixt, expreſs the quality of the lines, and thoſe of right, acute, obtuſe, the quantity of the ſpace contain'd between the ſaid lines.

An angle is right, when one of the lines is perpendicular to the other E D F

An angle is acute when its aperture is leſs than that of a right angle E D G

An angle is obtuſe when its aperture is greater than that of a right angle F D G

The middle letter D denotes the angle.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THE
Definition of a Surface.

A Surface is whatever has length and breadth without depth or thickneſs.

According to the ſentiments of the Geometricians, a Surface is a production of a line, juſt as a line is the production of a point; thus we are to imagine the line E F moving towards G H to conſtitute the ſurface E F G H, which is an extenſion bounded by lines, and has length and breadth without depth or thickneſs; this is commonly call'd a ſurface, but a figure if it be conſider'd in regard of its extremities, which are the bounding lines.

If the Surface be elevated or rais'd, 'tis ſaid to be Convex; but if depreſſed, ſunk in, or hollow, 'tis call'd a Concave; and if even and flat, a Plane. Thus

B is a Convex Surface.

C a Concave Surface.

A a Plane Surface.

D a Surface that is Convex, Concave and Plane.

This firſt part relates only to plane Surfaces.

The terminus, term or boundary of any thing is its extremity: thus a point is the terminus of a line, a line is the term of a ſurface, and a ſurface is the terminus of a body.

21
C
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B
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D
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A
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A
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f

Of Surfaces or Figures that are Rectilineal.

Surfaces take their particular names from the number of their ſides; thus

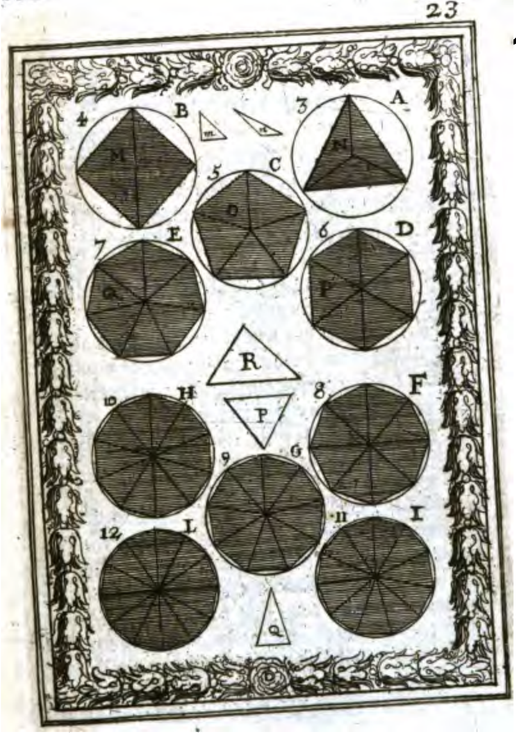
A IS a Trigon or triangle, a figure with three ſides.
B a Tetragon or ſquare, a figure of four ſides.
C a Pentagon, or a figure of five ſides.
D an Hexagon, a figure of ſix ſides.
E an Heptagon, or figure of ſeven ſides.
F an Octagon, or figure of eight ſides.
G an Enneagon, or figure of nine ſides.
H a Decagon, or figure of ten ſides.
I an Hendecagon, or figure of eleven ſides.
L a Dodecagon, or figure of twelve ſides.

All theſe figures are alſo call'd by the general name of Polygons.

Of TRIANGLES.

Triangles are diſtinguiſhed by the quality of their angles, and by the diſpoſition of their ſides. Thus

M is a right-angled Triangle, *i. e.* has one right angle.
N an obtuſe-angled Triangle, *i. e.* has one obtuſe angle.
O an acute-angled Triangle, *i. e.* has all three angles acute.
P an equilateral Triangle, *i. e.* has its three ſides equal.
Q an Iſoſceles Triangle, *i. e.* has only two ſides equal.
R a Scalene Triangle, *i. e.* has all its three ſides unequal.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Q

Of Quadrilaterals or figures that have four ſides.

A IS a ſquare or figure that has its four ſides equal and four angles right.

B a Rectangle, by ſome improperly call'd a long ſquare, has all its angles right or equal, but its ſides unequal.

C a Rhombus is a Quadrilateral that has its four ſides equal, but not its four angles.

D a Rhomboid has the oppoſite angles and ſides equal, without being equiangular or equilateral.

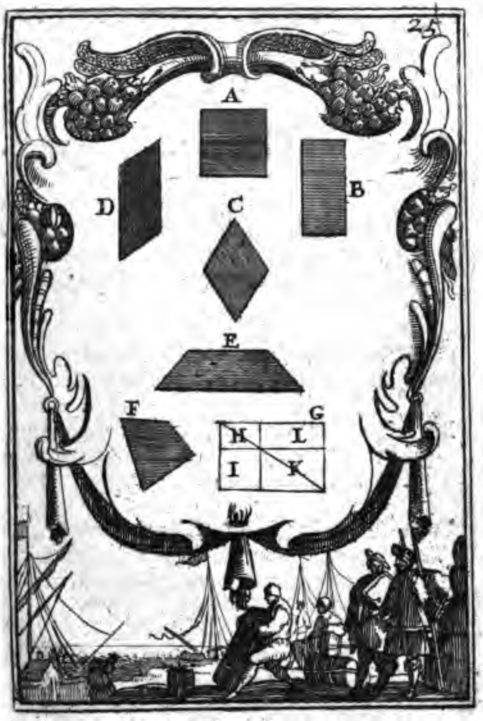
A B C D a Parallelogram is a Quadrilateral whoſe oppoſite ſides are parallel.

E a Trapezium has only two oppoſite ſides parallel, and the two others equal.

F a Trapezoid has its four ſides and angles unequal.

G if a Diagonal be drawn in a parallelogram, as alſo two lines parallel to the ſides, through the ſame point of the Diagonal, the parallelogram will be divided into four parallelograms; and three of them, *viz.* one of thoſe deſcribed upon the diameter and the two ſupplements, (*i. e.* the two parallelograms, which are not deſcribed about the diameter,) form a figure called a Gnomon; thus the three parallelograms H I L make a Gnomon, as do alſo the three parallelograms I K L.

All figures having more than four ſides, are call'd Polygonals or Multilaterals.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OF

Curves or Curvilineal figures.

A A Circle is a ſurface or figure perfectly round, deſcrib'd upon a center, from which the circumference in all its parts is equally diſtant.

a b c d. A Circumference is the extremity of a circle, or it is the circular line that bounds it.

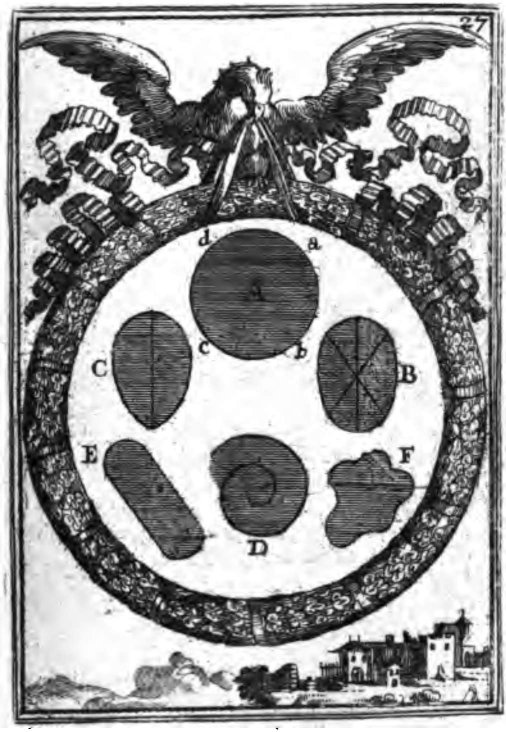
B an Oval is a curvilineal figure deſcrib'd upon ſeveral centers, and divided into two equal parts by all its diameters.

C an Ellipſe is alſo a curvilineal figure deſcrib'd upon ſeveral centers in the ſhape of an egg, and has but one diameter that divides it into two equal parts.

D a Volute or Scroll is a figure or ſurface bounded by a ſpiral line.

E is a Cylindric ſurface.

F is an irregular curvilineal figure, compos'd of ſeveral diſſimilar curve lines.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OF

Compound Figures.

A A Semicircle is a figure contain'd between half the circumference and the diameter.

B a portion of a Circle is a figure comprehended within any part of a circle and a right line.

g a large portion of a circle is greater than half the circle.

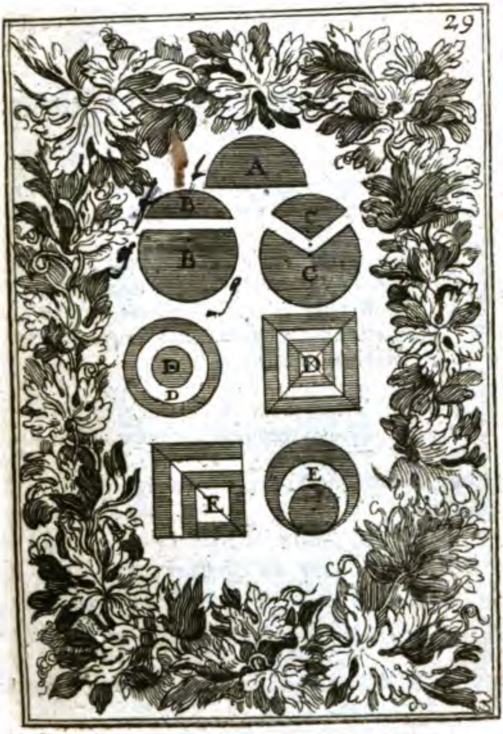
f a ſmall portion of a circle is that which is leſs than half the circle.

C a Sector is a figure contain'd between two ſemidiameters and an arc, greater or leſs than a ſemicircle.

There is alſo a large or ſmall Sector.

D Concentric figures are ſuch as have the ſame center.

E Excentric figures are ſuch as are deſcrib'd upon different centers.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OF

Regular and Irregular Figures.

A A Regular figure is that which has its oppoſite parts ſimilar and equal.

B an Irregular figure is ſuch an one as is compos'd of angles and ſides that are diſſimilar.

E E Similar figures are ſuch as have all their ſides proportional, though one may be greater, equal or leſs than another.

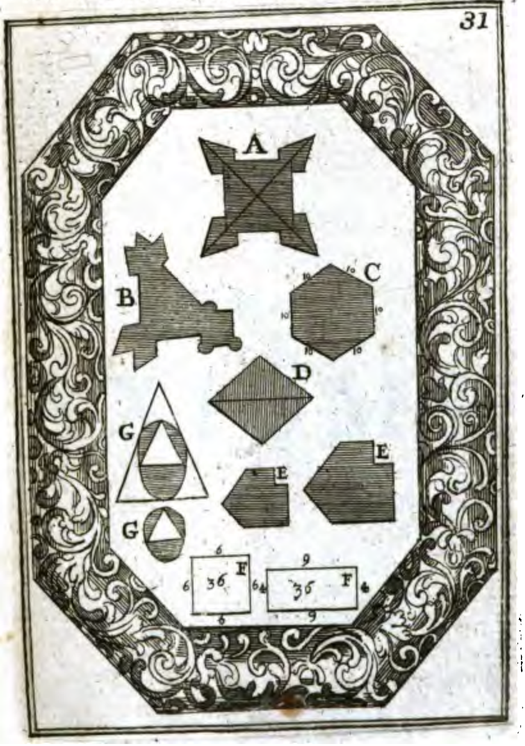
F F equal figures are ſuch whoſe contents are equal, though they may be ſimilar or diſſimilar.

C an Equiangular figure has all its angles equal.

E E one figure is ſaid to be ſimilar or equiangular to another, when all the reſpective angles of the one, are equal to all the reſpective angles of the other.

C D an Equilateral figure is one that has all its ſides equal.

G G Similar curvilineal figures are ſuch as will admit ſimilar Polygons to be inſcribed in them, or circumſcrib'd about them.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THE
AXIOMS.

AXIOMS.

I.

Things equal to the ſame third, are equal to one another.

The lines A C, A C, which are equal to A B, are alſo equal to one another.

II.

If to equal things, equal things be added, the whole will be equal.

The lines A C, A C are equal,
The lines C D, C D added are equal,
The whole A D, A D are alſo equal.

III.

If from equal things, equal things be taken away, the remainders will be equal.

If from the equal lines	A D, A D
you take away the equal lines	A C, A C
the remaining parts	C D, C D
will be alſo equal.	

IV.

If to unequal things, you add equal things, the whole will be unequal.

If to the unequal lines	D E, D E
you add the equal lines	A D, A D
the whole	A E, A E
will be equal.	

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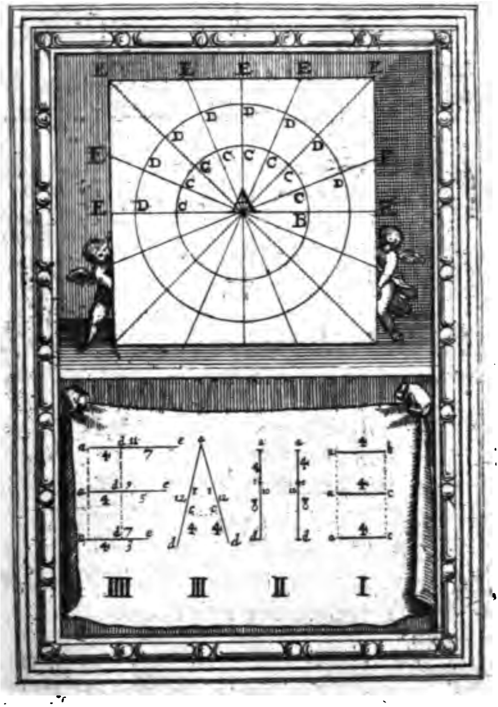
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V.

If from unequal things, equal things be taken, the remainders will be unequal.

If from the unequal lines, A E, A E
you take away the equals, A D, A D
the remainders, D E, D E
will be unequal.

VI.

Things double the ſame third, are alſo equal to one another.

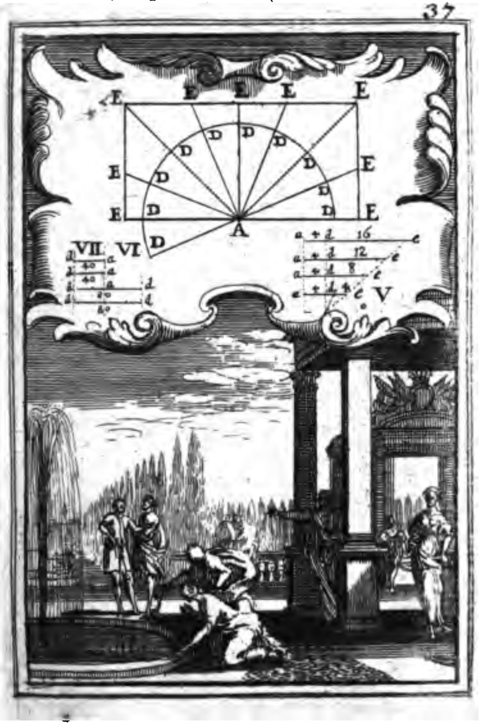
The right lines D D, D D
that are double the line A D
are equal among themſelves.

VII.

Things, that are halves of the ſame, or equal things, are alſo equal.

The lines A D, A D
which are the halves of the lines D D, D D
are equal to one another.

What has been ſaid of lines, may alſo be ſaid of numbers, ſurfaces and bodies.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80
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a + d 12 e
a + d 8 e
a + d 4 e
V

THE

PETITIONS.

THE

Petitions or Demands.

PETITION I.

Draw a right line from the point *A*
to the point *B*

OPERATION.

Apply a ruler to the points A & B
Draw the line demanded A B
by carrying the pencil along the ruler, and
cloſe to it from the point A
to the point B

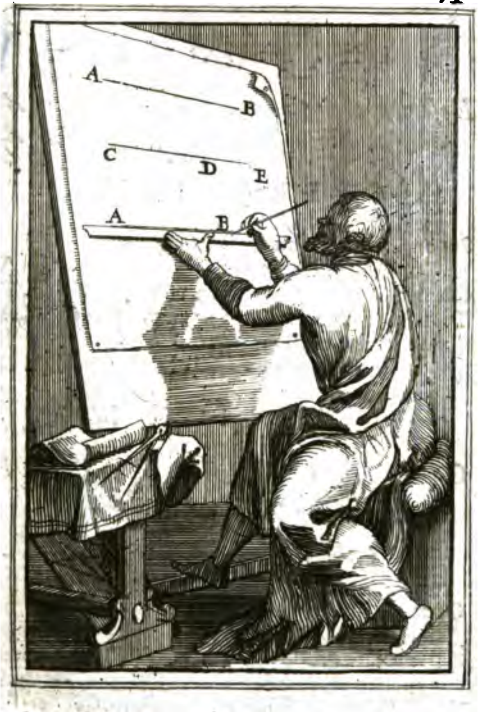
PETITION II.

Produce infinitely the line *C D*
on the ſide of the extremity *D*

OPERATION.

Join the ruler to the line C D
Continue infinitely that line C D
on the ſide of the extremity D
by carrying the pen along, cloſe to the ruler
towards E

41



PETITION III.

Deſcribe a circle upon the point A
and at the diſtance A B

OPERATION.

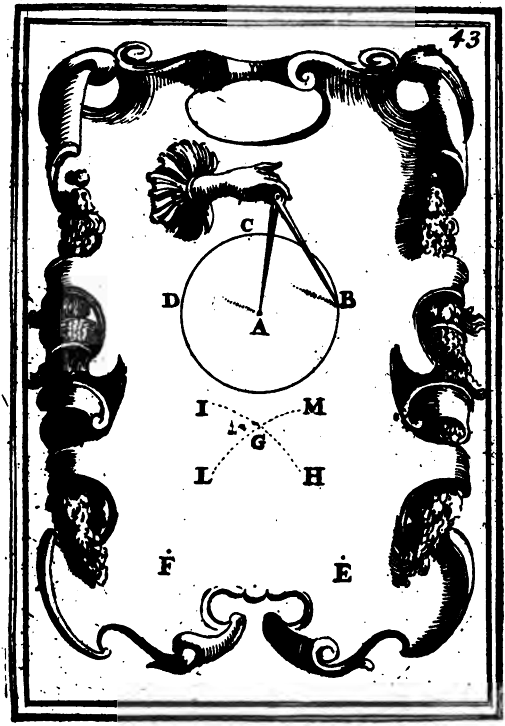
Set one of the points of the compaſs
upon the given point A
Open the other to the given point B
Turn the compaſſes about upon the point A
and trailing the point B
draw the circle demanded B C D

PETITION IV.

On the points E & F
make an interſection or ſection.

OPERATION.

Open the compaſſes at diſcretion, but ſo that the diſtance of the two points of the compaſſes may be greater than half the diſtance of the points propoſed E & F
With this diſtance of the compaſſes
Upon the point E deſcribe the arc L M
Upon the point F draw the arc H I
and the interſection requir'd will be G


43
C
D
B
A
I
M
G
L
H
F
E

THE
FIRST BOOK
OF THE
Deſcription of Lines.

BOOK the FIRST.

PROPOSITION I.

To erect a perpendicular upon the middle of a right line.

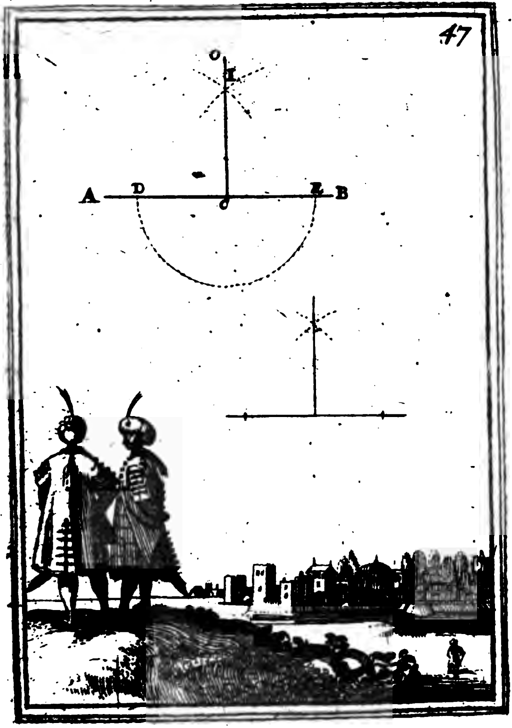
POSITION.

Let C be the point propoſed in the middle of the line A B, upon which the perpendicular is to be erected.

OPERATION.

UPon the given point	C
deſcribe at pleaſure the ſemicircle	D E
upon the points	D & E
make the ſection	I
from the point	C
draw the line demanded	C O
thro' the ſection	I

This line C O will be perpendicular to the line given A B, and erected upon the point propoſed C.


47
O
I
A
D
C
E
B

PROPOSITION II.

To erect a perpendicular upon the extremity of a right line propoſed.

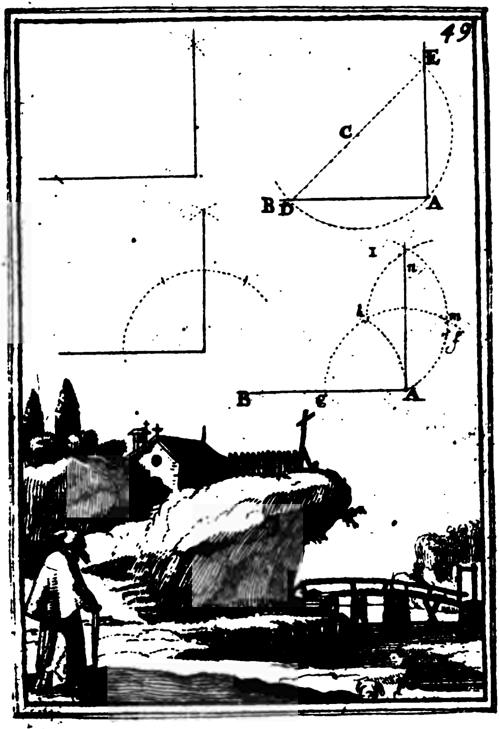
Let A be the extremity propoſed of the line A B, upon which the perpendicular is to be erected.

OPERATION.

Take at pleaſure the point	C
above the line	A B
from that point	C
with the diſtance	C A
Deſcribe the portion of the circle	E A D
Draw the right line	D C E
through the points	D & C
Draw the line demanded	A E
it will be perpendicular to	A B
and at the extremity propoſed	A

Another way.

Upon the point A deſcribe the arc	g h m
Upon the point g deſcribe the arc	A h
Upon the point h deſcribe the arc	A m n
Upon the point m deſcribe the arc	h n
Draw the line requir'd	A n


49
E
C
B
D
A
I
n
i
m
f
B
g
A

PROPOSITION III.

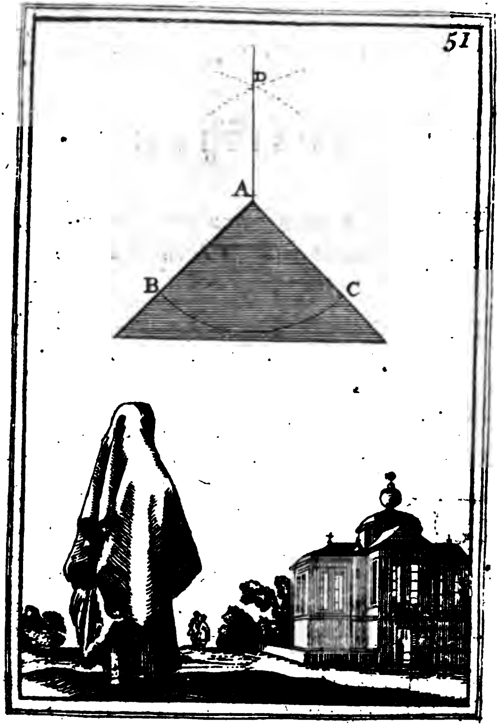
Upon an angle given to erect a right line that inclines neither to the right hand nor to the left.

Let B A C be the angle upon which the right line is to be raiſed, that inclines neither to the right hand nor to the left.

OPERATION.

UPon the angle given,	A
deſcribe at pleaſure the arc	B C
upon the extremities	B & C
make the ſection	D
from the point of the angle given	A
draw the line requir'd	A D
through the ſection	D

This right line	A D
ſhall be erected upon the angle	B A C
without inclining either to the right or left.	


51
D
A
B
C

PROPOSITION IV.

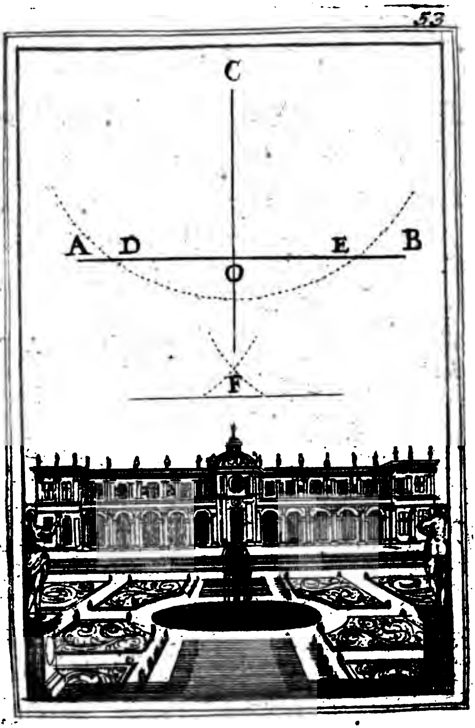
To let fall a perpendicular upon a given line, from a point without the line.

Let C be the point from which a line is to be let fall perpendicular to A B

OPERATION.

UPon the given point	C
deſcribe at pleaſure the arc	D E
cutting the line	A B
in the points	D & E
upon thoſe points	D & E
As centers make the ſection	F
draw the line	C F
and the line	C O
will be the line requir'd.	

* * *


53
C
A
D
E
B
O
F

PROPOSITION V.

Through a given point to draw a line parallel to a given right line.

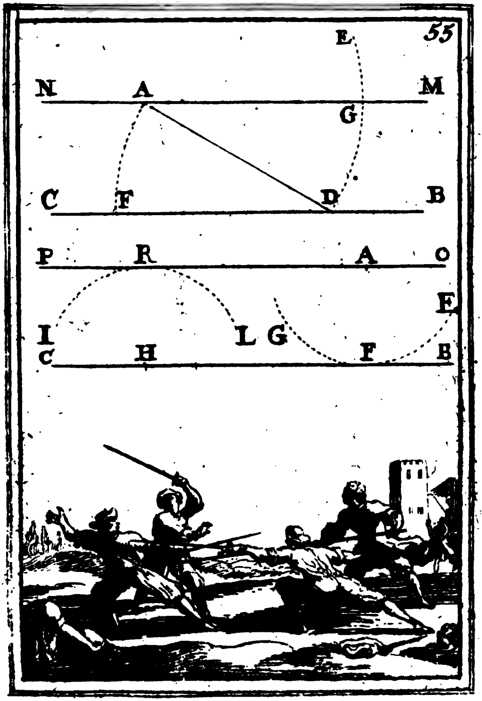
Let A be the given point thro' which a line is to be drawn parallel to the line B C.

OPERATION.

DRaw at pleaſure the oblique line	A D
upon the point	A
Deſcribe the arc	D E
upon the point	D
Deſcribe the arc	A F
make the arc	D G
equal to the arc	A F
Draw the line required	M N
thro' the points	A & G

Otherwiſe.

Upon the center A deſcribe the arc	E F G
touching the line	B C
without altering the legs of the compaſſes.	
Upon the point H deſcribe the arc	L R I
The point H is taken at pleaſure in the line	B C
Draw the demanded line	O P
thro' the point	A
and touching the arc	L R I


55
E
N
A
M
G
C
F
D
B
P
R
A
O
E
I
L
G
C
H
F
B

PROPOSITION VI.

To biſect a given finite right line.

POSITION.

Let A B be the right line propoſed to be divided into two equal parts.

OPERATION.

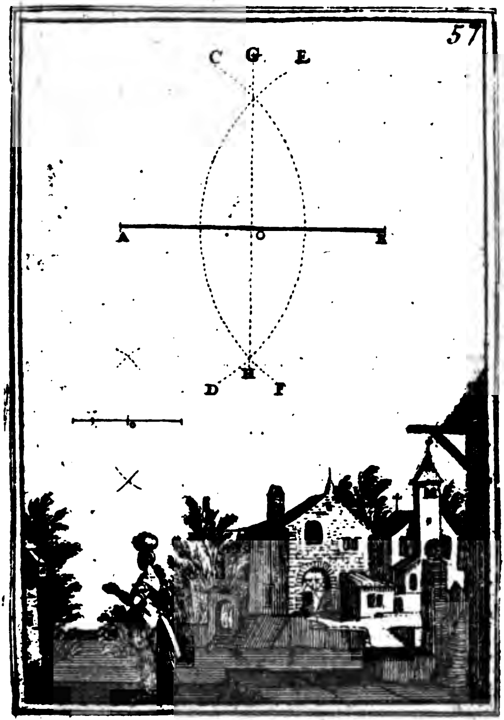
UPon the extremity A
as a center, deſcribe the arc C D

Without altering the diſtance of the legs of the compaſſes.

Upon the other extremity B
as a center, deſcribe the arc E F

Theſe arcs are to be made ſo as to interſect each other.

Draw the right line G H
through the interſections G & H,
A B then will be biſected at the point O


57
C
G
E
A
O
B
D
H
F

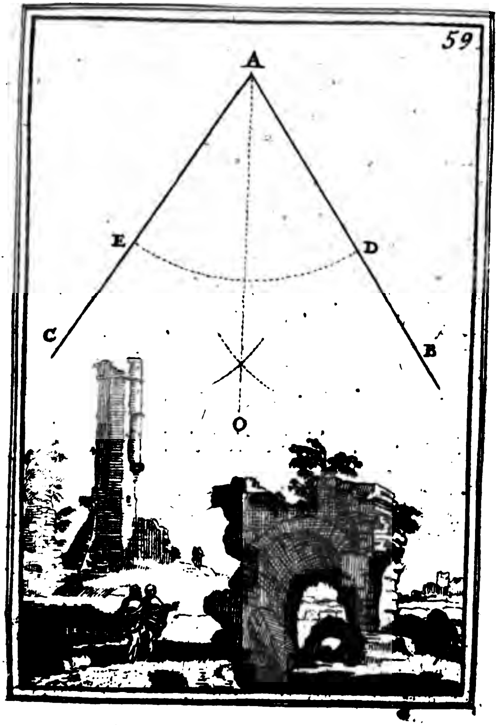
PROPOSITION VII.

To biſect a given rectilineal angle.

Let B A C be the angle propoſed to be biſected,

OPERATION.

Upon the angular point	A
deſcribe at pleaſure the arc	D E
upon the points	D & E
As centers make the ſection	O
draw the line	A O
This line	A O
will divide the given angle	B A C
into two equal parts.	


59
A
E
D
C
B
O

PROPOSITION VIII.

At the end of a given right line to make a rectilineal angle equal to a given rectilineal angle.

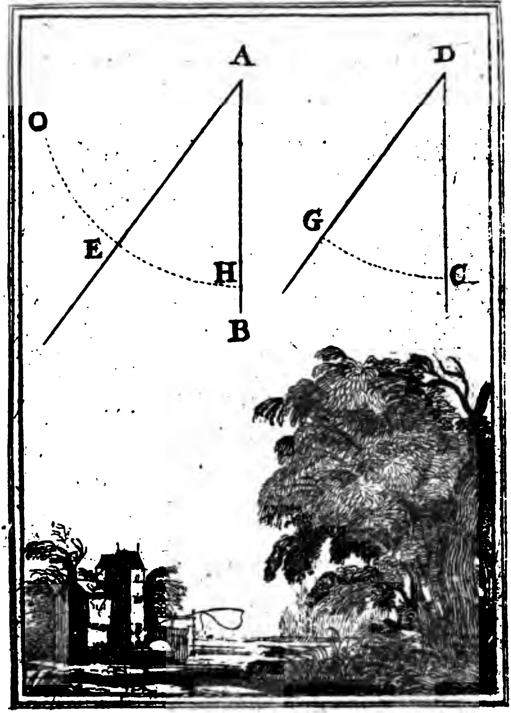
Let A be the end of the line A B, at which an angle is to be made equal to a given rectilineal angle C D G.

OPERATION.

UPon the angular point	D
deſcribe at pleaſure the arc	C G

Without altering the opening of the compaſſes.

Upon the extremity	A
deſcribe the arc	H O
Make the arc	H E
equal to the arc	C G
draw the line	A E
The angle	B A E
will be equal to the angle	C D G
which was the thing propoſed.	


A
D
O
G
E
H
C
B

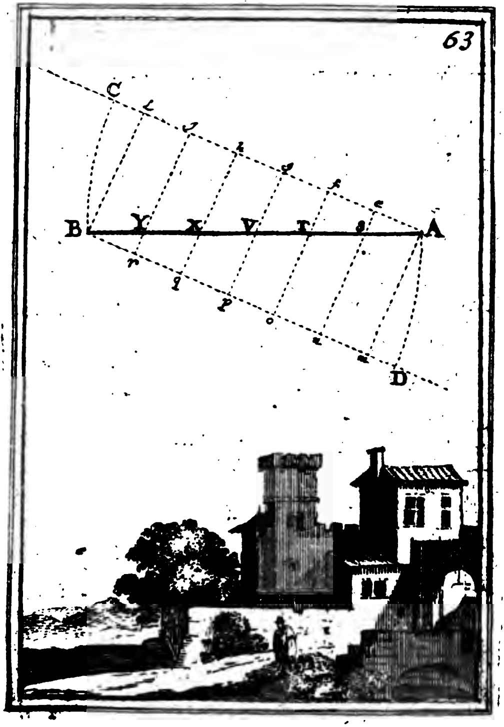
PROPOSITION IX.

To divide a given right line into any number of equal parts requir'd.

Let A B be the line propoſed to be divided into ſix equal parts.

OPERATION.

FRom the point	A
draw at pleaſure the line	A C
thro' the extremity	B
Draw the line	B D
parallel to the line	A C
from the points	A & B
and along the lines	A C, B D
Carry any ſix equal parts, *viz.*	e f g h I L
along the line	A C
R q p o n m along the line	B D
draw the lines	e n, f o, g p, h q, I R
Then the line	A B
will be divided into ſix equal parts at the ſections	S, T, V, X, Y


63
C
l
i
h
g
f
e
B
Y
X
V
T
S
A
r
q
p
o
n
m
D

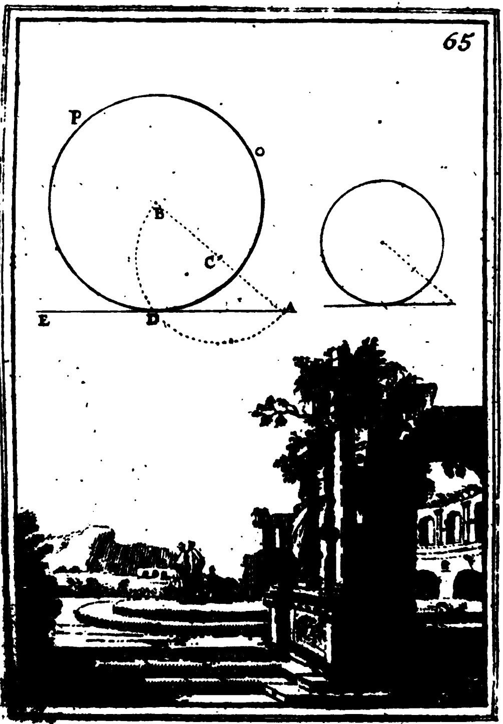
PROPOSITION X.

To draw a tangent to a circle propoſed through a given point.

Let A be the point thro' which the tangent to the circle D O P is to be drawn.

OPERATION.

FRom the center of the circle	B
draw the ſecant	B A
divide the line	B A
into two equal parts in	C
upon the point	C
with the radius	C A
Deſcribe the ſemicircle	A D B
cutting the circle in	D
from the given point	A
Draw the right line	A B
thro' the point	D
This right line	A B
will be the tangent requir'd.	


65
P
O
B
C
E
D
A

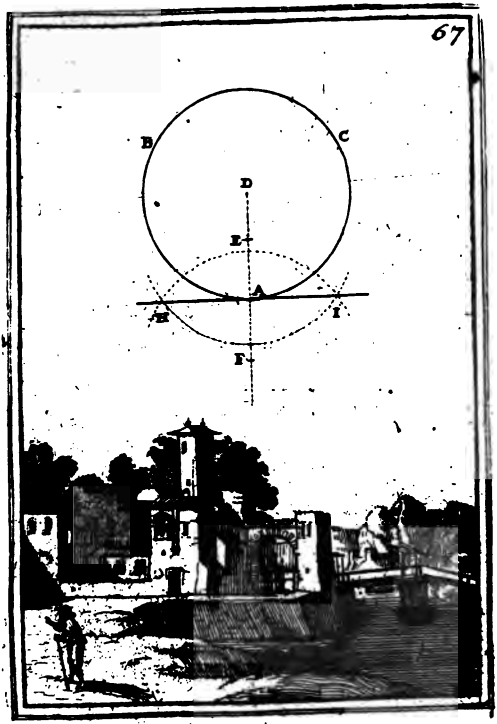
PROPOSITION XI.

To draw a right line that ſhall be a tangent to a circle at a given point.

Let A B C be the given circle, and the point of contact in its circumference A.

OPERATION.

FRom the point or center	D
draw the line	D F
thro' the point propoſed	A.
Thro' the point propoſed	A
and to the line	D F
draw the perpendicular	A H
continued towards	I
This tangent	H I
will touch the circle at the point	A
which was the thing required.	


67
B
C
D
E
A
H
I
F

PROPOSITION XII.

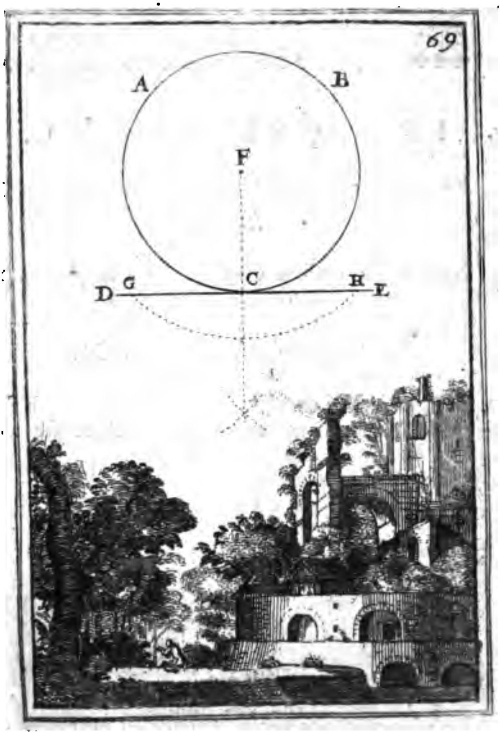
A circle and a right line that touches it being given to find the point of contact.

Let A B C be the circle to which the line G H is a tangent.

OPERATION.

Pag. 8. FRom the center of the circle F
let fall the perpendicular F C
upon the tangent D E

The ſection C
will be the point of contact ſought.


69
A
B
F
D
G
C
H
E

PROPOSITION XIII.

To draw a ſpiral line about a given right line.

Let I L be the line about which the ſpiral line is to be deſcribed.

OPERATION.

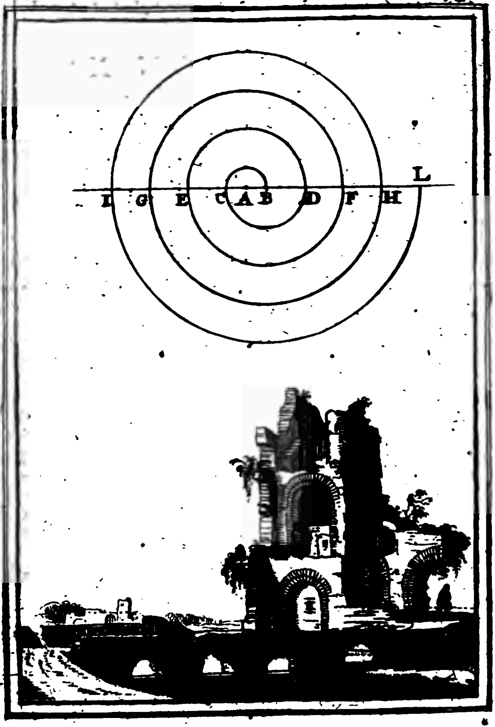
Pag, 18, DIvide half the right line I L
into as many equal parts as there are to be revolutions.

EXAMPLE.

To make one of four revolutions.

Pag, 11. Divide the half B I
into four equal parts B C E G I
Divide alſo B C
into two equal parts in A
upon the point A
Deſcribe the ſemicircles B C, D E, F G, H I
upon the point B
Deſcribe the ſemicircles C D, E F, G H, I L
and you will have the ſpiral line ſought.

71



PROPOSITION XIV.

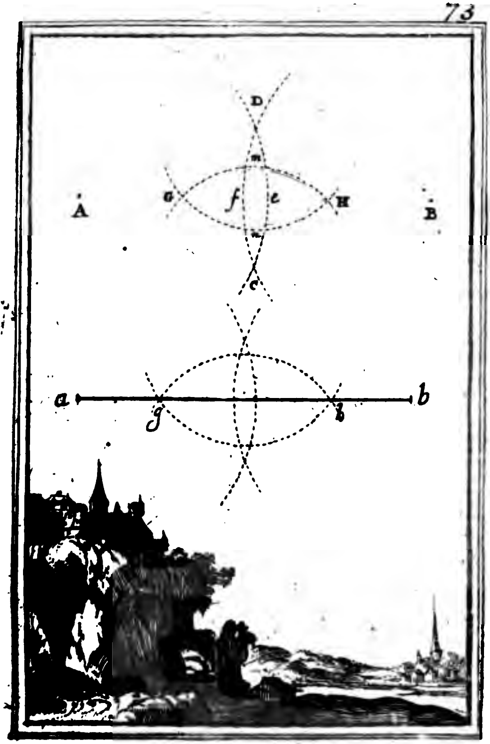
Between two given points to find two other directly interpoſed.

Let A and B be the points given, between which two others are to be found directly interpos'd, by the help of which a right line may be drawn from the point A to the point B, with a ſhort rule.

OPERATION.

UPon the points A & B
as centers, make the interſections C & D
upon the points C & D
As centers make the interſections G & H

Theſe points G & H
are the points requir'd, by the aſſiſtance of which a right line may be drawn from the point A to the point B, which could not be done at once with a rule leſs than the length between A & B.


D
m
A
G
f
e
H
B
c
a
b
g
h

THE
SECOND BOOK,
OF THE
Conſtruction of plane Figures,

BOOK the SECOND.

PROPOSITION I.

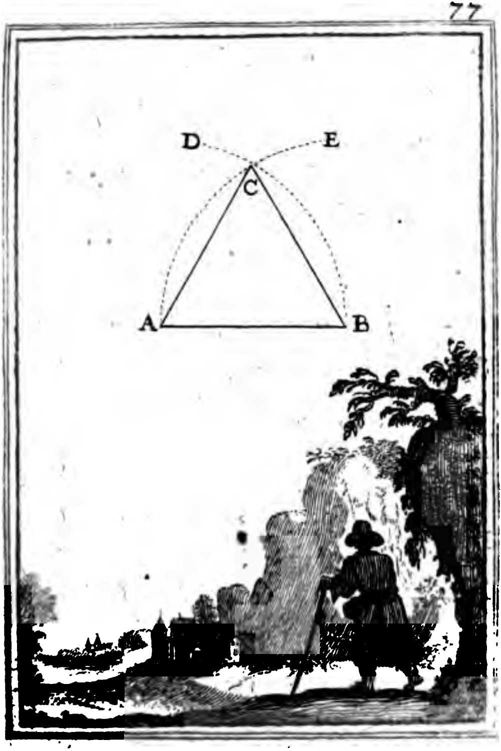
To make an equilateral triangle upon a given line.

Let AB be the given line upon which the equilateral triangle is to be conſtructed.

OPERATION.

UPon the extreme point	A
with the radius	A B
Deſcribe the arc	B D
upon the extremity	B
with radius	B A
Deſcribe the arc	A B
from the interſection	C
Draw the lines	C A, C B

A B C will be the equilateral triangle required.


D
E
C
A
B

PROPOSITION II.

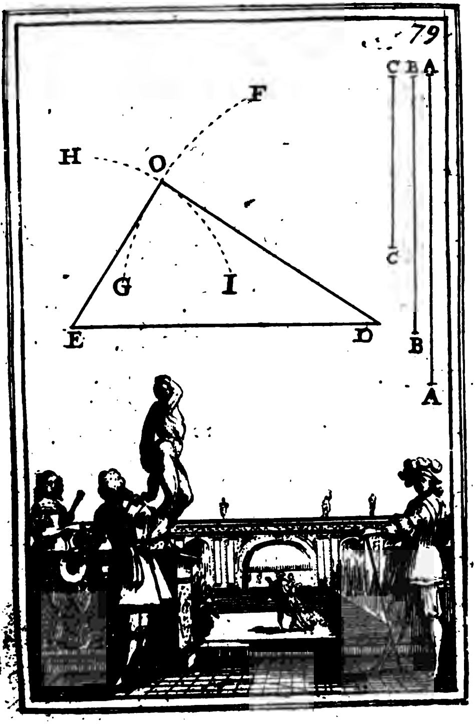
To make a triangle whoſe three ſides are equal to three given right lines.

Let A B C be the three given lines; a triangle is to be made whoſe three ſides are equal to them.

OPERATION.

DRaw the right line	D E
equal to the line	A A
upon the point	D
with the radius	B B
Deſcribe the arc	G F
upon the point	E
with the radius	C C
Deſcribe the arc	H I
from the interſection	O
Draw the lines	O E, O D

The triangle D E O will be compoſed of three ſides equal to the three lines given A A, B B, C C.


79
C
B
A
F
H
O
G
I
E
D
C
B
A

PROPOSITION III.

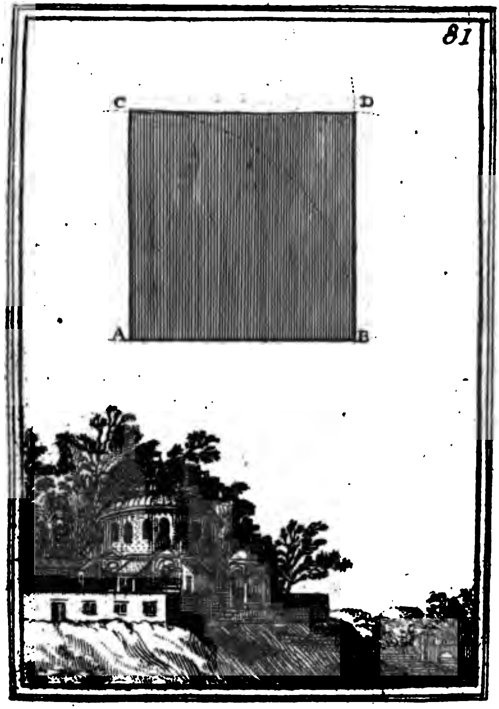
To make a ſquare upon a given right line.

Let A B be the given right line, upon which the ſquare is to be made.

OPERATION.

Pag. 50. Erect the perpendicular	A C
upon the point	A
As a center, deſcribe the arc	B C
upon the points	B & C
with the radius	A B
Make the ſection	D
from the point	D
Draw the lines	D C, D B

A B C D will be the ſquare requir'd to be conſtructed upon the given right line A B.


81
C
D
A
B

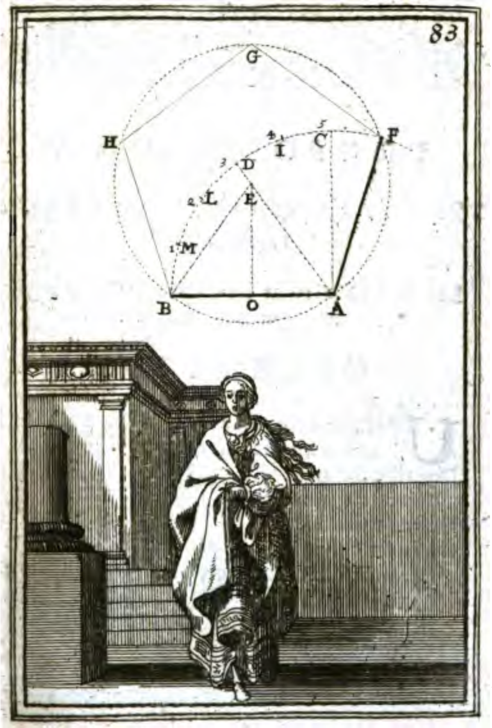
PROPOSITION IV.

To make a regular pentagon upon a given right line.

Let A B be the given line, upon which the pentagon is to be conſtructed.

OPERATION.

	UPon the extremity	A
	and with the radius	A B
	Deſcribe the arc	B D F
Page 50.	Erect the perpendicular	A C
	Divide the arc	B C
	into five equal parts	I D L M
	Draw the right line	A D
Page 58.	Divide the baſe	A B
	into two equal parts in	O
Page 46.	Erect the perpendicular	O E
	upon the interſection	E
	with the radius	E A
	Deſcribe the circle	A B F G H
	Carry round five times, the line	A B
	in the circumference of the circle, and a regular equiangular equilateral pentagon, will be compleated.	


83
G
H
F
5
C
4
I
3
D
2
L
E
1
M
B
O
A

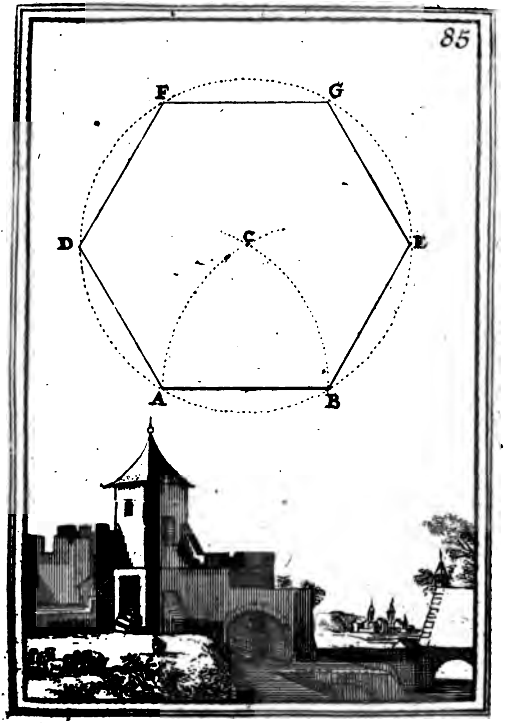
PROPOSITION V.

To make a regular hexagon upon a given right line.

Let A B be a right line, upon which a regular hexagon is to be made.

OPERATION.

UPon the extremities	A & B
and with the radius	A B
Deſcribe the arcs	A C, B C
upon the ſection	C
Deſcribe the circle	A B E F G
Carry ſix times the line given	A B
in the circumference, and you will have a	
regular hexagon	A B E F G D
upon the given line	A B


85
F
G
D
C
E
A
B

PROPOSITION VI.

Upon a given right line to deſcribe any polygon from an hexagon to a dodecagon.

Let A B be a line upon which an hexagon, heptagon, or octagon, &c. is to be made.

OPERATION.

Pag. 58. Biſect the line A B in the point O
Pag. 46. erect the perpendicular O I
upon the point B deſcribe the arc A C
Divide A C into ſix equal parts M, N, P, Q, R

This is to be done, if an heptagon is to be made.

Upon the point C with the interval
of one part C M
deſcribe the arc M D
D will be the center for deſcribing a circle capable of containing ſeven times the line given.

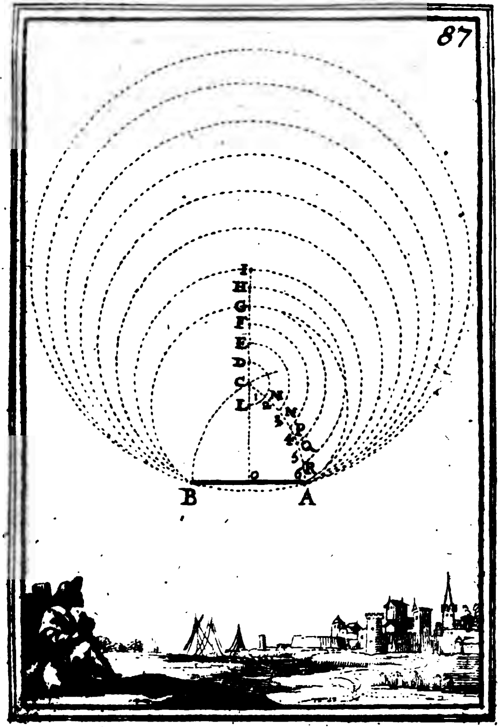
For an octagon.

Upon the center C, with the interval
of two parts C N
Deſcribe the arc N E
E will be the center of a circle capable of containing eight times the given line A B

For an heptagon.

Take three parts C P

and ſo for the reſt adding one part.


87
I
H
G
F
E
D
C
L
1
2
3
4
5
6
N
N
P
Q
R
o
B
A

PROPOSITION VII.

To make a polygon of any number of ſides from twelve to twenty four, upon a given right line.

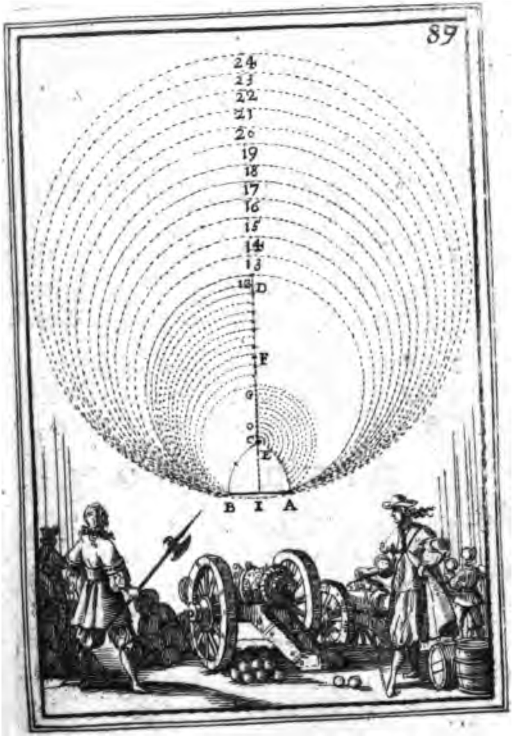
Let A B be the line, upon which the polygon is to be made.

OPERATION.

Divide the arc A C
into twelve equal parts from the point C.
Take as many of the parts of C A
as the number of the ſides of the polygon is above twelve.

Example.

If you would deſcribe a polygon of fifteen ſides.
Upon the point C
with the radius of three of theſe parts C E
deſcribe the arc E O.
A C of twelve, C O of three together make fifteen.
Upon the point O with the radius O B
deſcribe the arc B F
Upon the point F with the radius F A
deſcribe a circumference, and it will contain the line given A B
twelve times.
and ſo alſo for any other polygon.


89
24
23
22
21
20
19
18
17
16
15
14
13
12
D
F
C
E
B
I
A

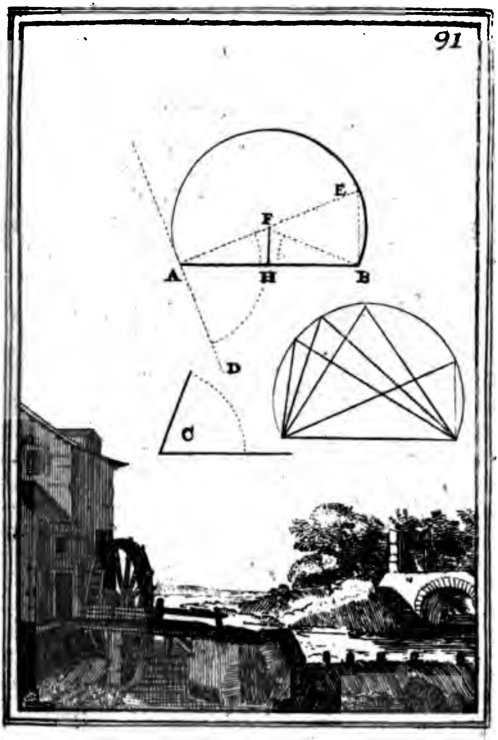
PROPOSITION VIII.

To deſcribe a portion of a circle capable of containing an angle equal to an angle given, upon a given right line.

Let A B be the right line, upon which a portion of a circle capable of containing an angle equal to the given angle is to be deſcribed C

OPERATION.

Page 62.	Make the angle	B A D
	equal to the angle	C
Page 50.	Erect upon	A D
	the perpendicular	A E
Page 58.	Biſect the line	A B
	in the point	H
Page 46.	Erect the perpendicular	H F
	upon the ſection	F
	with the radius	F A
	Deſcribe the portion of the circle	A E B
	All the angles you make in this ſegment of the circle, and upon the given line	A B
	will be equal to the angle	C


91
E
F
A
H
B
D
C

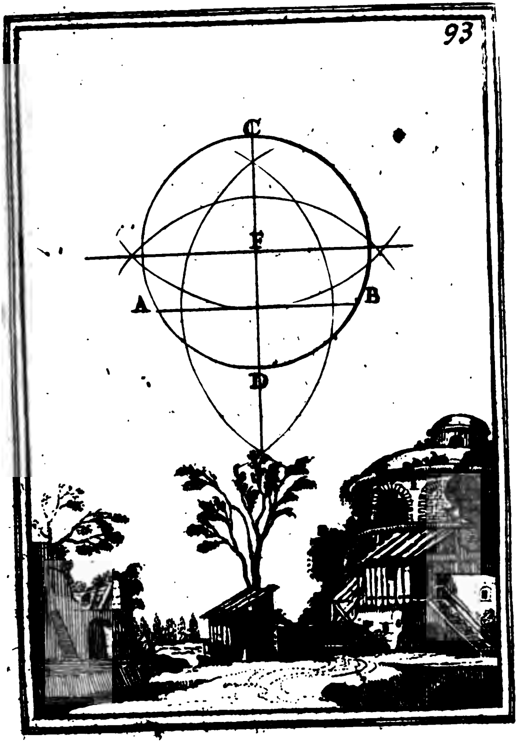
PROPOSITION IX.

To find the center of a given circle.

Let A B C be the circle propoſed, whoſe center is to be found.

OPERATION.

Draw at pleaſure the right line A B
terminating in the circumference A B C
Page 58. Biſect the right line A B
by the line D C
Page 58. Biſect alſo the right line C D
in the point F
The point F will be the center of the circle required A B C


93
C
E
A
B
D

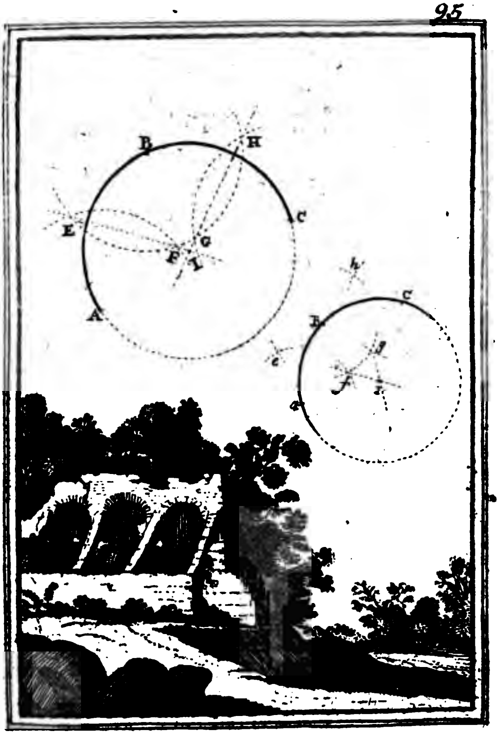
PROPOSITION X.

To compleat the circumference of a circle whoſe center is loſt.

Let A B C be the part of the circumference given, whoſe center is to be found, in order to the finiſhing the circle.

OPERATION.

Take at pleaſure the three points	A B C
in the circumference begun upon the points	A & B
Make the ſections	E & F
Draw the right line	E F
upon the points	B & C
Make the ſections	G & H
draw the right line	G H
upon the interſection and center	I
and with the interval	I A
compleat the circumference begun.	


B
H
C
E
G
F
I
A
h
c
b
g
e
f
i
a

PROPOSITION XI.

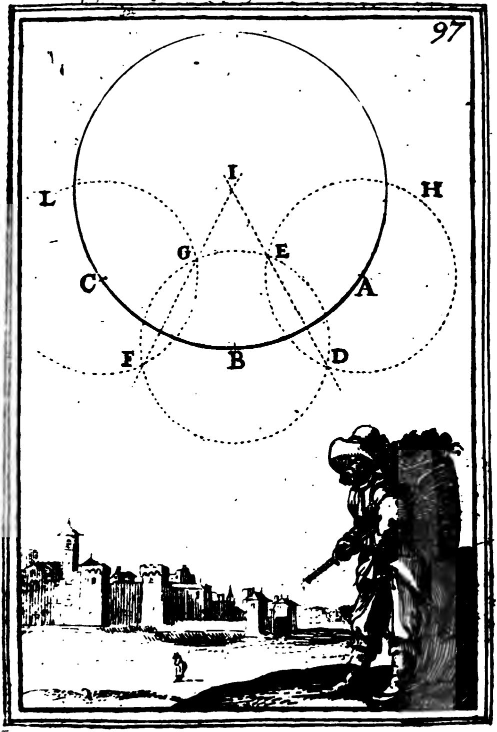
To deſcribe a circle that ſhall paſs thro' three given points.

Let A, B, C, be the three points thro' which the circle is to paſs.

OPERATION.

UPon the points given A, B, C,
deſcribe three circles DEH, DEF, FGL
with the ſame radius, and interſecting at the
points D & E, F & G
Draw the right lines DE, FG
till they meet in I
upon the point I
with the radius IA
Deſcribe the circle requir'd.

This operation is ſimilar to the preceding.


97
I
L
H
G
E
C
A
F
B
D

PROPOSITION XII.

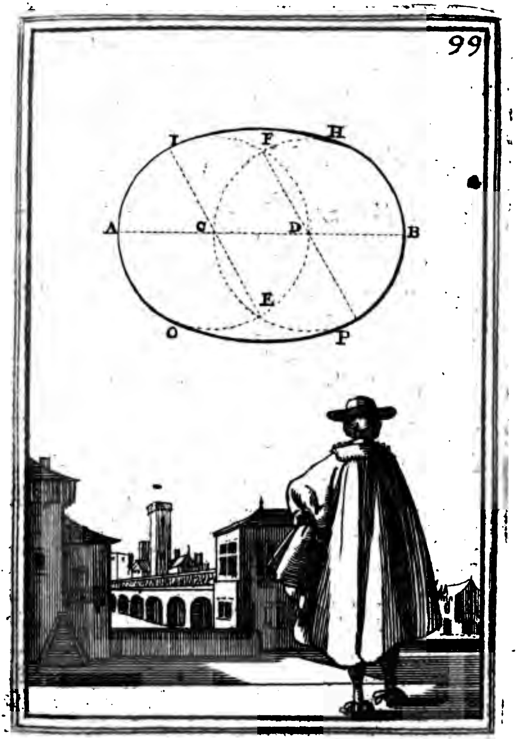
To deſcribe an oval upon a given length.

OPERATION.

Let A B be the length upon which the oval is to be made.

Page 64. Divide the length given	A B
into three equal parts	A C D B
upon the points	C & D
with the radius	C A
Deſcribe the circles	A E F, B E F
upon the interſections	E & F
and with the diameter	E H
As a radius, deſcribe the arcs	I H, O P

A I H B P O will be the oval requir'd.


99
I
F
H
A
C
D
B
E
O
P

PROPOSITION XIII.

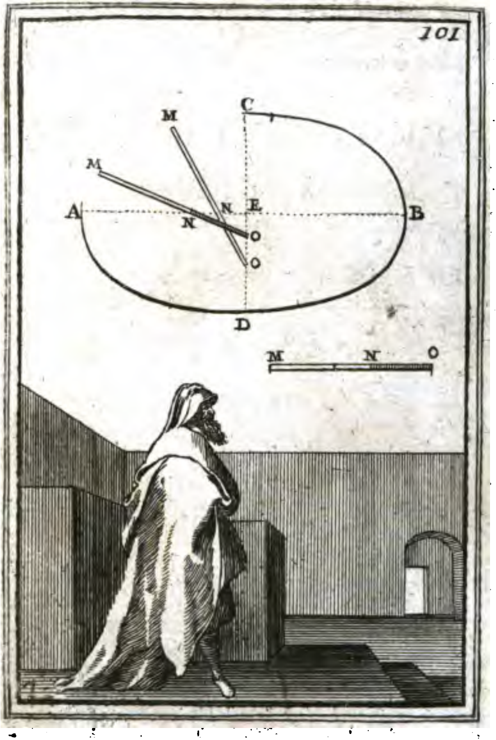
To deſcribe an oval upon two given diameters.

Let A B, C D be the diameters upon which the oval is to be conſtructed.

OPERATION.

MAke the ruler M O
equal to the greater ſemidiameter A E.
upon which mark the length M N
equal to the leſſer ſemidiameter C E
This ruler being thus diſpos'd.
Place it after ſuch a manner upon the diameters A B, C D
that the point N
ſliding along the line A B
the extremity O
may always be in the line C D
carrying along thus the rule M O
Deſcribe the oval with the extremity M

* * *


101
C
M
M
A
N
N
E
B
O
O
D
M
N
O

PROPOSITION XIV.

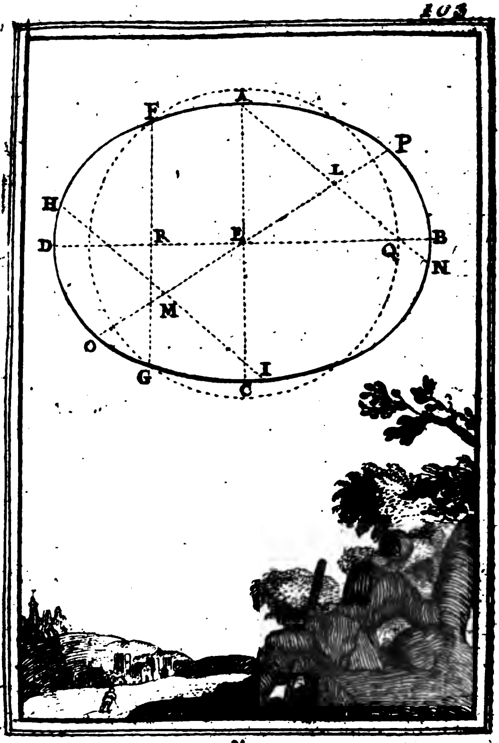
To find the center and the two diameters of an oval.

Let A B C D be the oval propoſed, whoſe center and diameters are to be found.

OPERATION.

	IN the oval propoſed	A B C D
Pag. 56.	draw at pleaſure	
	the two parallel lines	A N, H I
Page 56.	Biſect the lines	A N, H I
	in the points	L & M
	Draw the line	P L M O
Page 58.	Biſect it in	E
	and the point E will be the center	
	upon the point	E
	Deſcribe at pleaſure the circle	F G Q
	cutting the oval in	F & G
	thro' the interſections	F & G
	Draw the right line	F G
Page 58.	Biſect it in	R
	Draw the greateſt diameter	B D
	thro' the points	E R
	Thro' the center	E
Page 56.	Draw the leaſt diameter	A E C
	parallel to the line	F G
	and what was propoſed will be effected.	

103



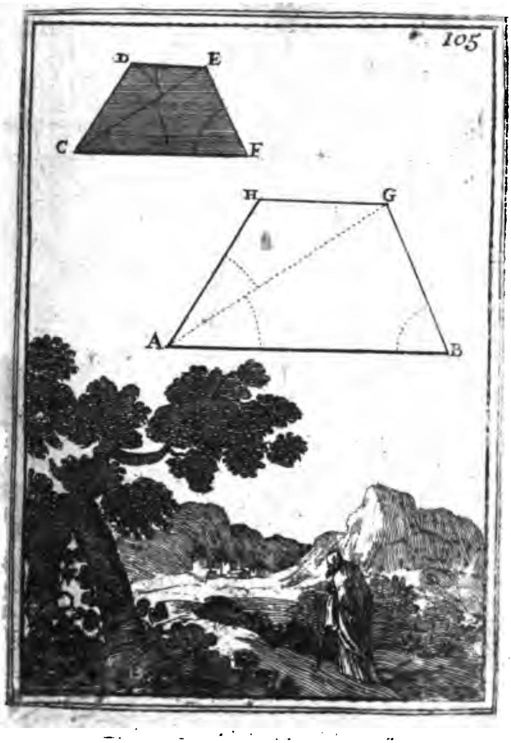
PROPOSITION XV.

To make a rectilineal figure upon a given right line ſimilar to a given rectilineal figure.

Let A B be the line upon which a figure ſimilar to the figure C D E F is to be drawn.

OPERATION.

	Draw the diagonal	C E
Page 62.	make the angle	A B G
	equal to the angle.	C F E
Page 62.	Make the angle	B A G
	equal to the angle	F C E
	the triangle	A B G
	will be ſimilar to the triangle	C F E
	after the ſame manner,	
age 62.	Make the triangle	A G H
	ſimilar to the triangle	C E D
	The whole figure	A B G H
	will be ſimilar to the whole figure	C D E F


105
D
E
C
F
H
G
A
B

THE
THIRD BOOK,
OF THE
Inſcribing of FIGURES.

BOOK the THIRD.

PROPOSITION I.

To inſcribe in a given circle, an equilateral triangle, hexagon or dodecagon.

Let A C D be the circle in which an equilateral triangle, &c. is to be inſcribed.

OPERATION.

For an equilateral triangle.

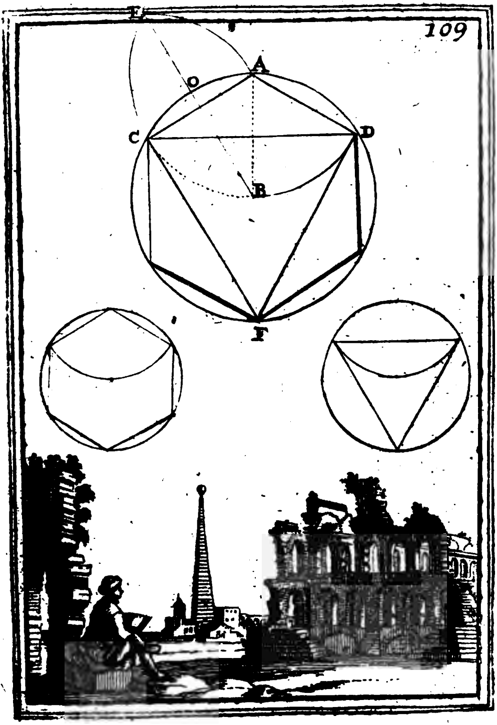
UPon a point as	A
with the interval of the ſemidiameter	A B
Deſcribe an arc	C B D
Draw the right line	D C
Carry that diſtance	C D
from the point	C
to the point	F
Draw the lines	F C, F D
The triangle requir'd will be	C D F

For an hexagon.

Carry round ſix times the ſemidiameter in the given circumference.	A B

For a dodecagon.

Page 58. Biſect the arc of the hexagon	A C
in the point	O
the ſide of the dodecagon will be	A O


109
E
A
O
C
D
B
F

PROPOSITION II.

To inſcribe a ſquare or octagon in a given circle.

Let AB CD be the circle in which the ſquare or octagon is to be inſcribed.

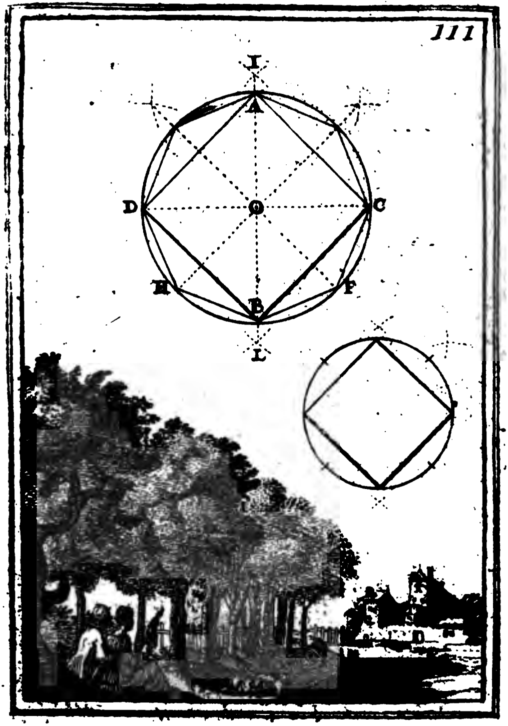
OPERATION.

For a ſquare.

Draw the diameters AB, CD
interſecting each other at right
angles, that is, draw the right line C D
thro' the center of the circle O
upon the points or extremities C & D
Make the interſections I & L
Draw the right line I L
paſſing thro' the center O
Theſe lines or diameters A B, C D
will interſect at right angles.
Draw the lines AC, AD, BC, BD, & ACBD
will be the ſquare requir'd.

For an octagon.

Page 58. Subdivide each quarter of the circle into two equal parts, and you will have an octagon.


111
I
A
D
O
C
H
B
F
L

PROPOSITION III.

To inſcribe a pentagon or decagon in a given circle.

Let A B C D be the circle propoſed.

OPERATION.

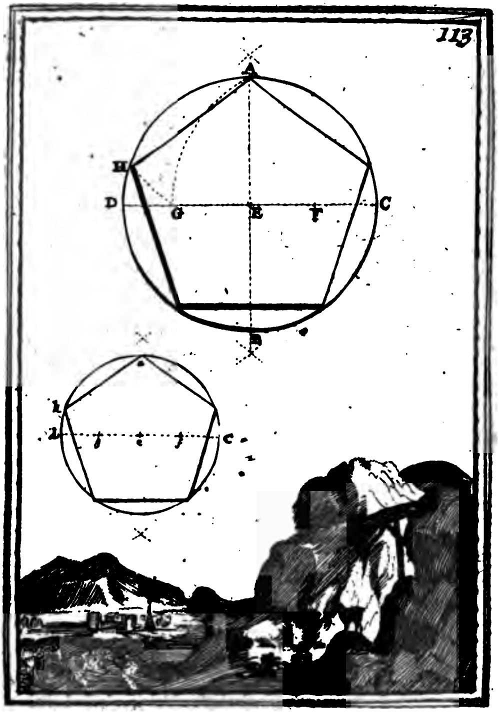
For a pentagon.

DRaw the two diameters AB, C D
Page 58. interſecting each other at right angles in E
Biſect the ſemidiameter C E
in the point F.
Upon the point F
as a center, with the radius F A
Deſcribe the arc A G
upon the point A
with the radius A G
Deſcribe the arc G H.
The right line A H
will divide the circle into five equal parts,

For the decagon.

Page 58. Subdivide each part of the circle into two equal parts.

2


113
A
H
D
G
E
F
C
B
a
h
d
f
e
f
c

PROPOSITION IV.

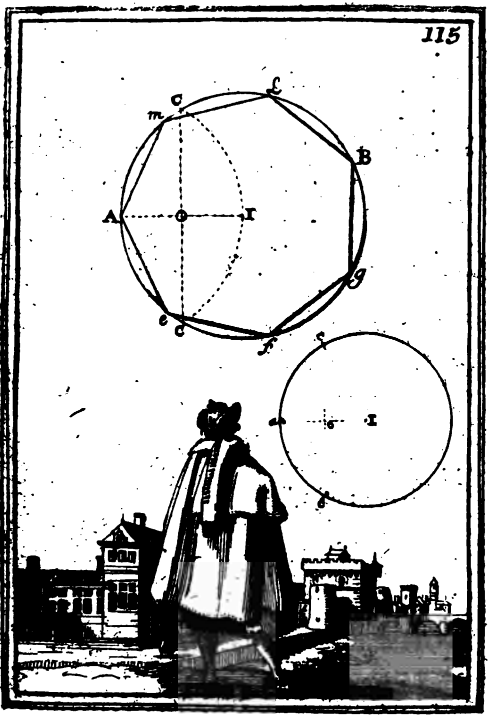
To inſcribe an heptagon in a given circle.

Let A B C be the circle propoſed in which the heptagon is to be inſcribed.

OPERATION.

DRaw the radius	I A
upon the extremity	A
with the radius	A I
Deſcribe the arc	C I C
Draw the right line	C C
Carry the half	C O

ſeven times in the circumference of the circle, and the heptagon requir'd will be inſcribed.


115
c
m
L
B
A
o
I
g
e
c
f
s
a
o
I
b

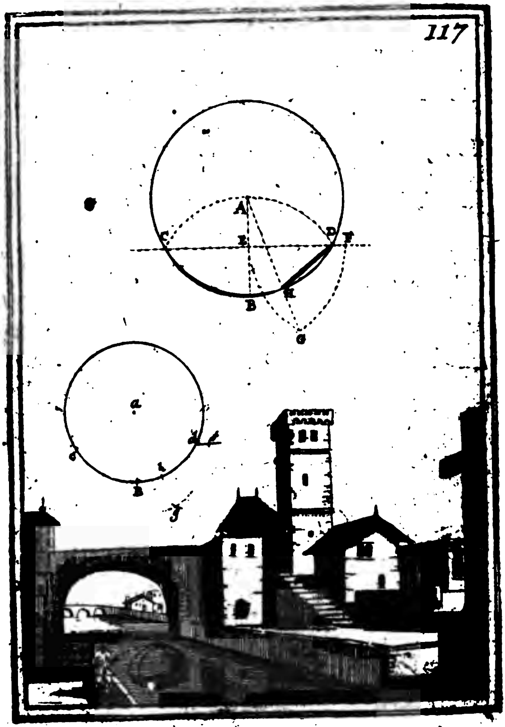
PROPOSITION V.

To inſcribe an enneagon in a given circle.

Let B C D be the circle in which the enneagon is to be inſcrib'd.

OPERATION.

DRaw the radius	A B
upon the extremity	B
with the diſtance	B A
Deſcribe the arc	C A D
Draw the right line	C D
produced towards	F
Make the line	E F
equal to the line	A B
upon the point	E
Deſcribe the arc	F G
upon the point	F
Deſcribe the arc	E G
Draw the line	A G
Then the ninth part of the circumference will be	D H


117
A
C
D
F
E
H
B
G
a
d
e
c
b
B
f

PROPOSITION VI.

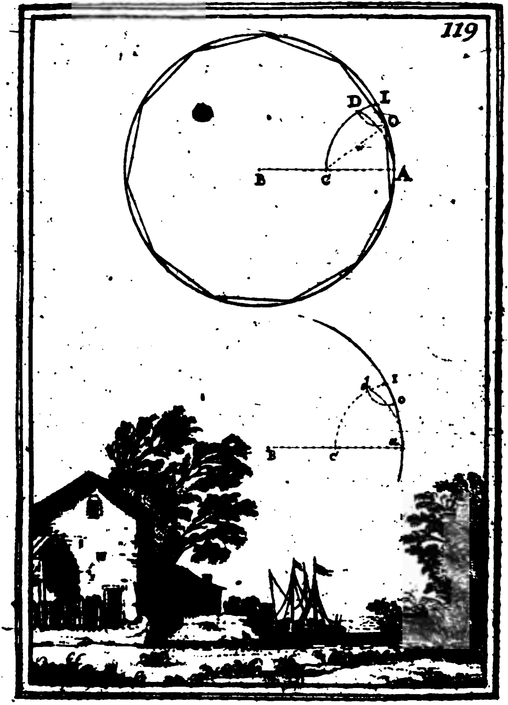
To inſcribe an hendecagon in a given circle.

Let AEF be the given circle in which the hendecagon is to be inſcrib'd.

OPERATION.

DRaw the radius	A B
Page 58. Biſect the radius	A B
in the point	C
upon the points	A & C
with the diſtance	A C
Deſcribe the arcs	C D I, A D.
upon the point	I
with the diſtance	I D
Deſcribe the arc	D O
the diſtance	C O

will be the ſide of the hendecagon exact enough for practice.


119
D
I
O
B
C
A
I
O
B
C

PROPOSITION VII.

To inſcribe in a given circle any polygon you pleaſe.

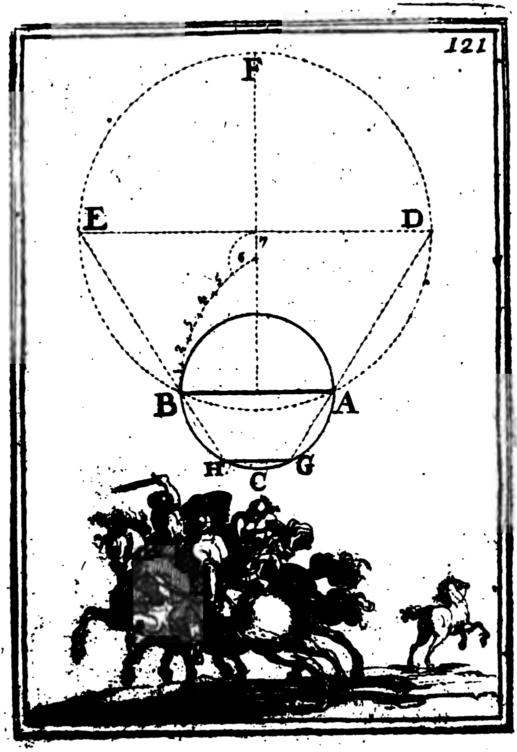
Let B A C be the circle in which you would have an heptagon inſcribed.

OPERATION.

Page 84. 86. 88. 90. DRaw the diameter A B
describe the circle A B F
capable of containing ſeven times A B

After the ſame way as if you would make upon AB a polygon ſimilar to that which is to be inſcribed in the given circle A B C

Page 56. Draw the diameter D E
parallel to the diameter A B
Draw the right lines D A G, E B H
thro' the extremities D A, E B
G H, will divide the circle given A B C
into ſeven equal parts.
After the ſame manner act in other polygons.


121
F
E
D
7
6
B
A
H
G
C

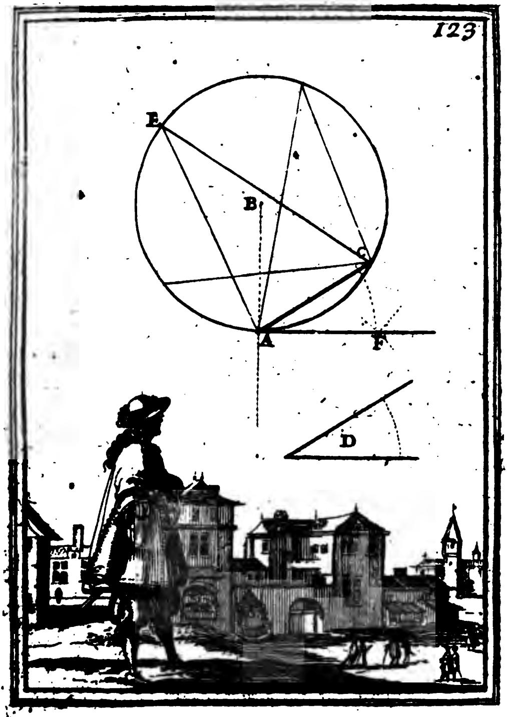
PROPOSITION VIII.

To cut off a ſegment of a given circle, capable of an angle equal to any rectilineal angle propoſed.

Let ACE be the given circle, a portion of which is to be cut off, capable of an angle equal to the angle D

OPERATION.

	Draw the ſemidiameter	A B
Page 66.	Draw the tangent	A F
Page 62.	Make the angle	F A C
	equal to the given angle	D
	All the angles made upon	A C
	in the ſegment	A E C
	will be equal to the given angle	D
	Therefore the portion	A E C
	is the ſegment requir'd.	


123
E
B
C
A
F
D

PROPOSITION IX.

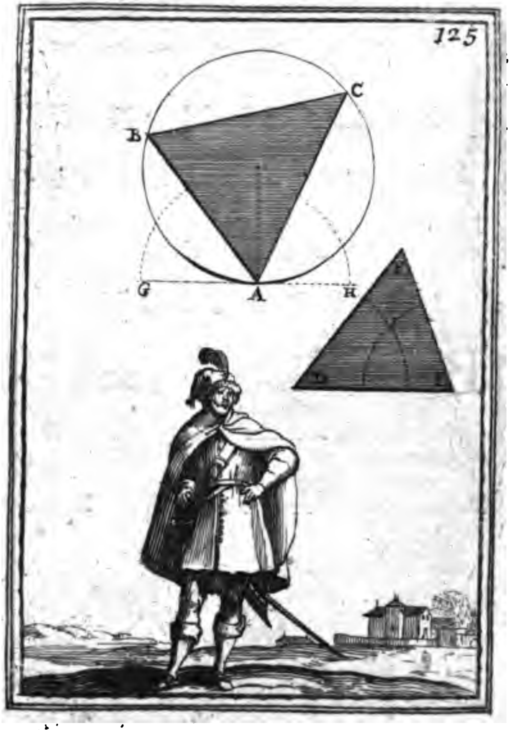
To inſcribe in a circle a triangle ſimilar to a given triangle.

Let A B C be the circle in which a triangle, ſimilar to the triangle D E F is to be inſcribed.

OPERATION.

Page 66.	Draw the tangent	G H
	upon the point of contact	A
Page 62.	Make the angle	H A C
	equal to the angle	E
Page 62.	Make alſo the angle	G A B
	equal to the angle	D
	Draw the line	B C

A B C will be the triangle requir'd to be ſimilar to the triangle given D E F


125
C
B
G
A
H

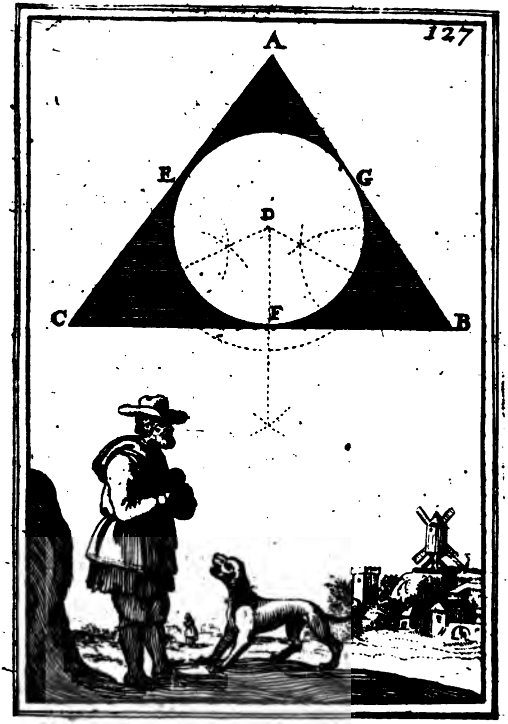
PROPOSITION X.

To inſcribe a circle in a given triangle.

Let A B C be the triangle in which the circle is to be inſcribed.

OPERATION.

Page 60.	Biſect the angles	B & C
	by the right lines	BD & CD
	from the interſection	D
Page 54.	Let fall the perpendicular	D F
	Upon the center	D
	with the diſtance	D F
	Deſcribe the circle requir'd	E F G


127
A
E
G
D
C
F
B

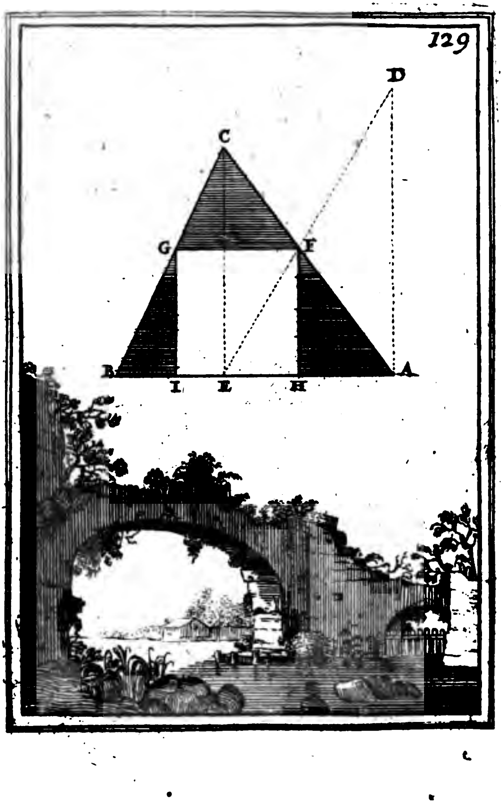
PROPOSITION XI.

To inſcribe a ſquare in a given triangle.

Let A B C be the triangle in which the ſquare requir'd is to be inſcribed.

OPERATION.

Page 50.	Erect the perpendicular	A D
	upon the extremity of the baſe	A B
	Make this perpendicular	A D
	equal to the baſe	A B
	from the angle	C
Pag. 56.	Draw the line	C E
	parallel to the line	A D
	Draw the oblique line	D E
	thro' the ſection	F
Page 56.	Draw the line	F G
	parallel to the baſe	A B
Page 56.	Draw the lines	F H, G I
	parallel to the line	C E
	And the ſquare requir'd will be	F G H I



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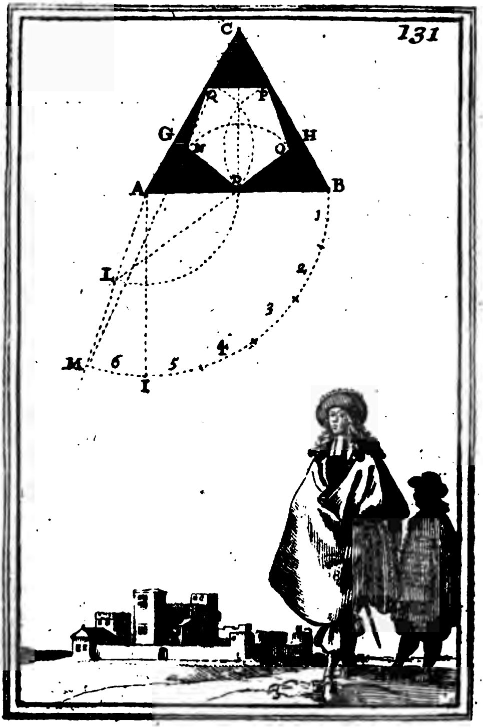
PROPOSITION XII.

To inſcribe a regular pentagon in an equilateral triangle.

Let A B C be the triangle in which the pentagon is to be inſcribed.

OPERATION.

Page 54.	LET fall the perpendicular	A I
	upon the center	A
	Deſcribe the arc	B I M
	Divide into five equal parts the arc	B I
	Carry on the ſixth	I M
	Draw the line	A M
Page 58.	Divide	A M
	into two equal parts in	L
	Upon the point	A
	deſcribe the arc	L D
	Draw the right line	L D to H
	Make the part	A G
	equal to the part	B H
	Draw the right lines	D G, M C
	upon the center	D
	with the diſtance of the ſection	N
	Deſcribe the arc	N O
	upon the points	N O
	Deſcribe the arcs	D Q, D P
	Draw the lines	O P, P Q, N Q
	And the pentagon demanded ſhall be	D O P Q N


131
C
Q
P
G
H
N
O
A
D
B
1
2
3
4
5
6
L
M
I

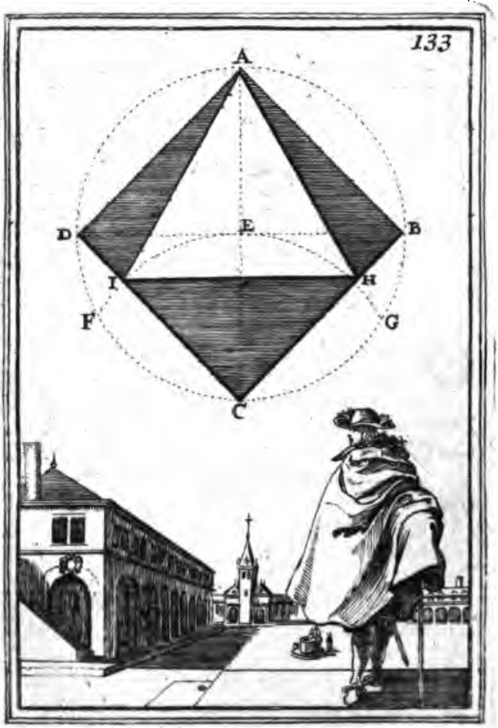
PROPOSITION XIII.

To inſcribe an equilateral triangle in a ſquare.

Let A B C D be the ſquare in which the equilateral triangle is to be inſcribed.

OPERATION.

Draw the diagonals	A C, B D
upon the center	E
and with the diſtance	E A
Deſcribe the circle	A B C D
upon the point	C
with the diſtance	C E
Deſcribe the arc	G E F
Draw the right lines	A F, A G
Draw the right line	H I
The equilateral triangle requir'd is	A H L


133
A
E
D
B
I
H
F
G
C

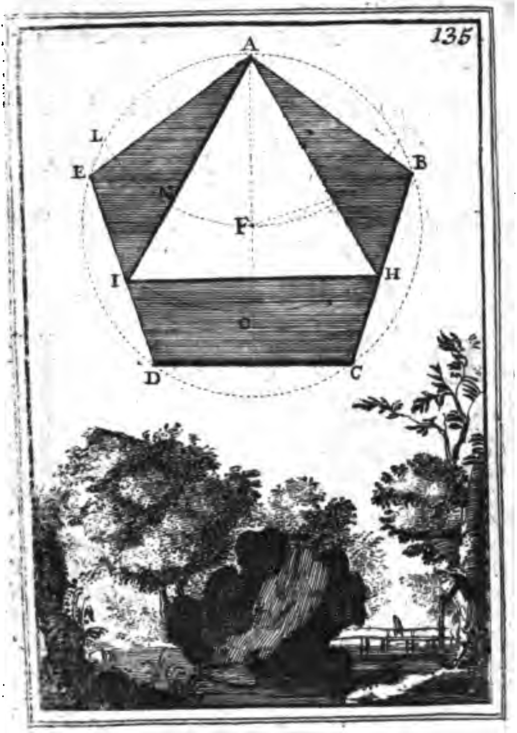
PROPOSITION XIV.

To inſcribe an equilateral triangle in a pentagon.

Let A B C D E be the pentagon in which an equilateral triangle is to be inſcribed.

OPERATION.

Page 98. Circumſcribe the circle	A B C D E
upon the point	A
and with the diſtance of the radius	A F
Deſcribe the arc	F L
Cut that arc	F L
into two equal parts in	N
Draw the line	F N I
upon the point	A
with the diſtance	A I
Deſcribe the arc	H O I
draw the lines	A H, H I
The triangle demanded will be	A H I


135
A
L
E
B
N
F
I
H
O
D
C

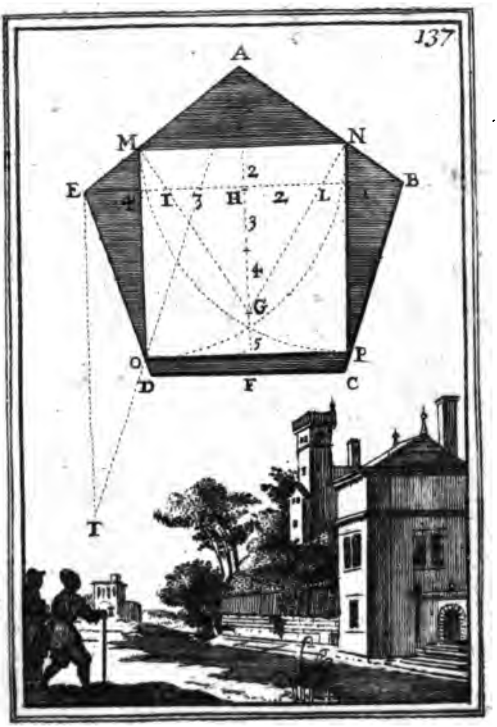
PROPOSITION XV.

To inſcribe a ſquare in a pentagon.

Let ABCDE be the pentagon in which a ſquare is to be inſcribed.

OPERATION.

	Draw the line	B E
Page 54.	let fall the perpendicular	E T
	from the extremity of	B E
	Make this perpendicular	E T
	equal to the line	B E
	Draw the line	A T
	thro' the ſection	O
Page 56.	Draw the line	O P
	parallel to the ſide	C D
	On the extremities	O & P
Page 50.	Erect the perpendiculars	O M, P N
	Draw the line	N M
	The ſquare requir'd will be	N M O P


137
A
M
N
E
B
2
4
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3
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2
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O
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C
T

THE
FOURTH BOOK,
OF THE
Circumſcription of Figures.

BOOK the FOURTH.

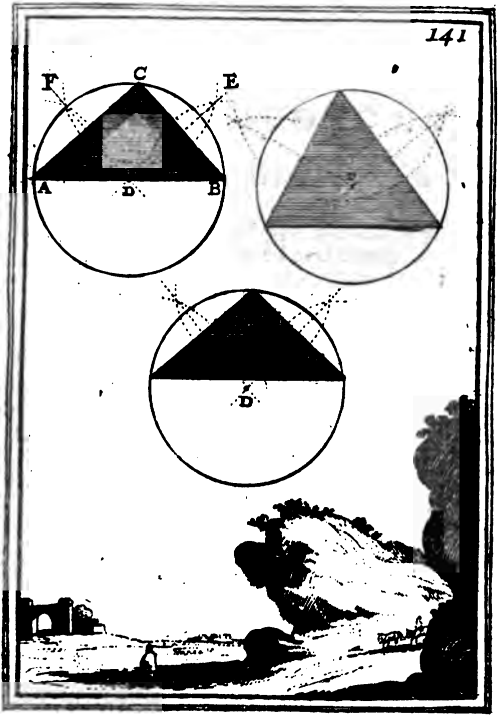
PROPOSITION I.

To circumſcribe a circle about a given triangle.

Let A B C be the triangle about which the circle is to be circumſcribed.

OPERATION.

Page 98. DEſcribe the circumference A B C thro' the three points A, B, C and the thing required will be done.


141
F
C
E
A
D
B
D

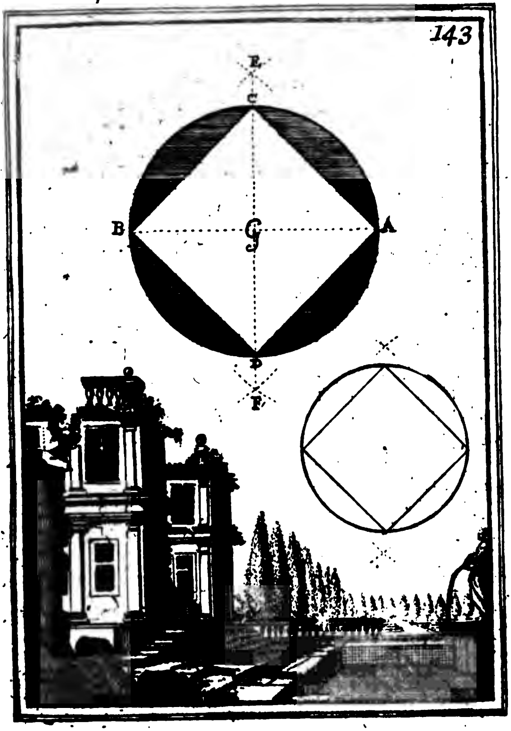
PROPOSITION II.

To circumſcribe a circle about a ſquare.

Let A B C D be the ſquare about which the circle is to be circumſcribed.

OPERATION.

Draw the two diagonals	A B, C D
upon the interſection or center	G
with the diſtance	G A
Deſcribe the circle demanded	A B C D


143
E
C
B
G
A
D
F

PROPOSITION III.

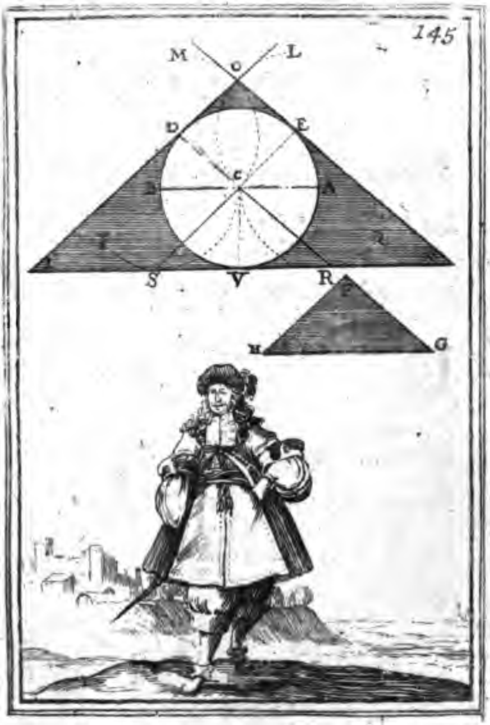
To circumſcribe a triangle ſimilar to a given triangle, about a given circle.

Let D E V be the circle, about which a triangle, ſimilar to the triangle F G H, is to be deſcribed.

OPERATION.

	DRaw the diameter	A B
	thro' the center	C
Pag. 62.	Make the angle	A C E
	equal to the angle	H
Page 62	Make the angle	B C D
	equal to the angle	G
	Produce the lines	E C, D C
	towards	R & S
Page 56.	Draw the tangent	N O
	parallel to the line	D R
Page 56.	Draw the tangent	O I
	parallel to the line	E S
Page 56.	Draw alſo the tangent	N I
	parallel to the diameter	A B

I N O will be the triangle requir'd ſimilar to the triangle F G H, and circumſcribed about the circle D E V.


145
M
O
L
D
E
C
B
A
T
Q
S
V
R
F
H
G

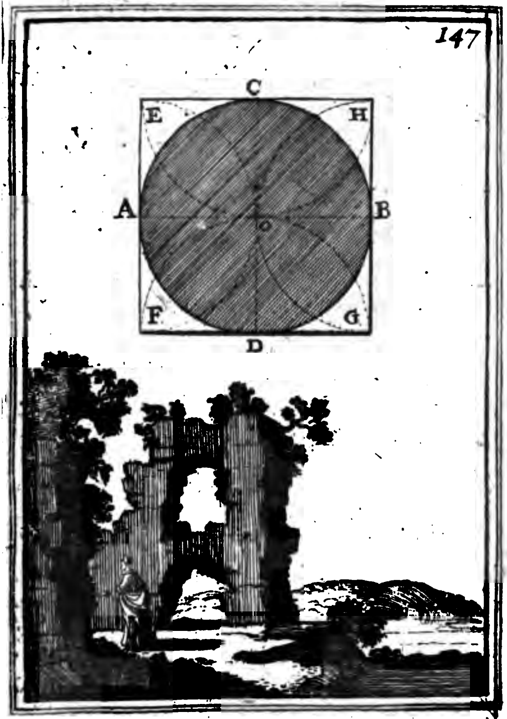
PROPOSITION IV.

To circumſcribe a ſquare about a circle.

Let A B C D be the circle about which a ſquare is to be circumſcribed.

OPERATION.

DRaw the diameters	A B, C D.
interſecting each other at right angles in O	
upon the points	A, C, B,
with the diſtance	A
Deſcribe the ſemicircles	H O G, H O
	E O F, F O
Draw the right lines	E F, F G, G H,
thro' the interſections	E, F, G,
The ſquare demanded will be	E F G


147
C
E
H
A
B
O
F
G
D

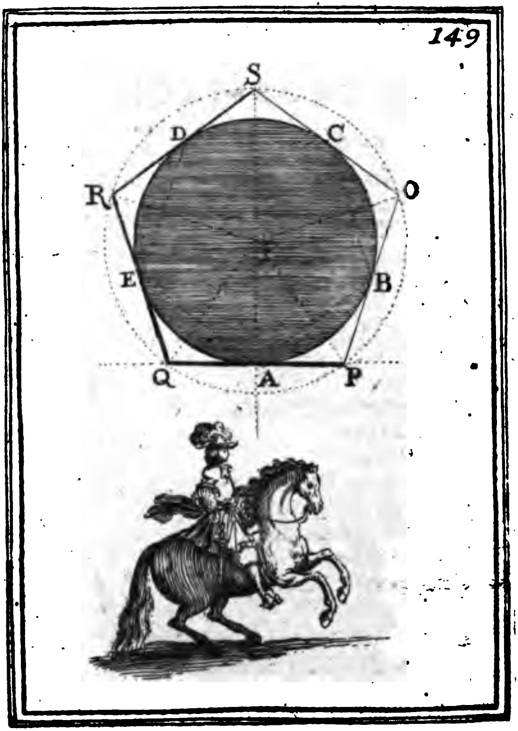
PROPOSITION V.

To circumſcribe a pentagon about a given circle.

Let A B C D E be the given circle about which a pentagon is to be circumſcribed.

OPERATION.

Pag. 124.	INſcribe the pentagon	A B C D E
	upon the center	F
	and thro' the middle of each ſide	
	Draw the lines	F O, F P, F Q, F R, F S
	Draw the line	F A
Page 68.	Draw the tangent	P Q
	thro' the point	A
	Upon the center	F
	with the radius	F P
	Deſcribe the circle	O P Q R S
	Draw the ſides of the pentagon demanded thro' the ſections	O, P, Q, R, S


149
S
D
C
R
O
E
B
Q
A
P

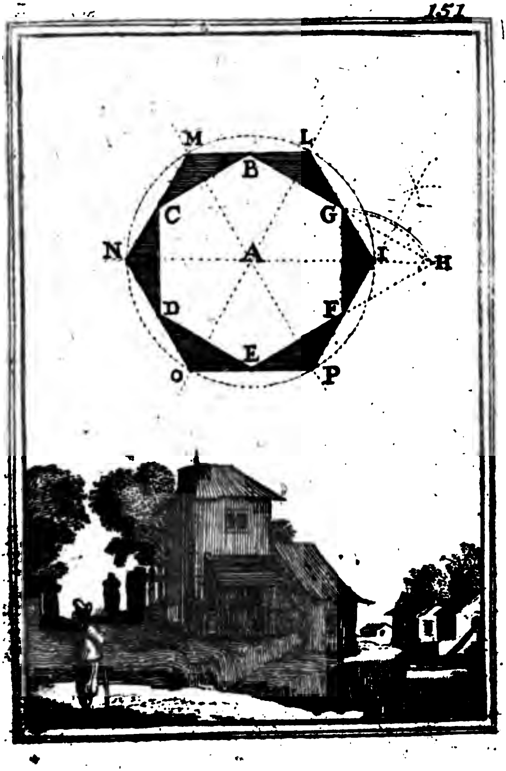
PROPOSITION VI.

To circumſcribe a regular polygon about another of the ſame ſort.

Let BCDEFG be the polygon given, about which another ſimilar polygon is to be circumſcribed.

OPERATION.

	Produce two ſides as	BG, EF
	until they meet in	H
	Draw the line	AH
Page 60.	Draw the line	FI
	biſecting the angle	GFH
	upon the center	A
	with the diſtance	AI
	Deſcribe the arc	IMO
	Draw the radius's	AL, AM, AN, AO
	thro' the middle of each ſide	
	Draw the ſides of the exteriour polygon demanded, thro' the ſections	I, L, M, N, O, P


M
L
B
C
G
N
A
I
H
D
F
E
O
P

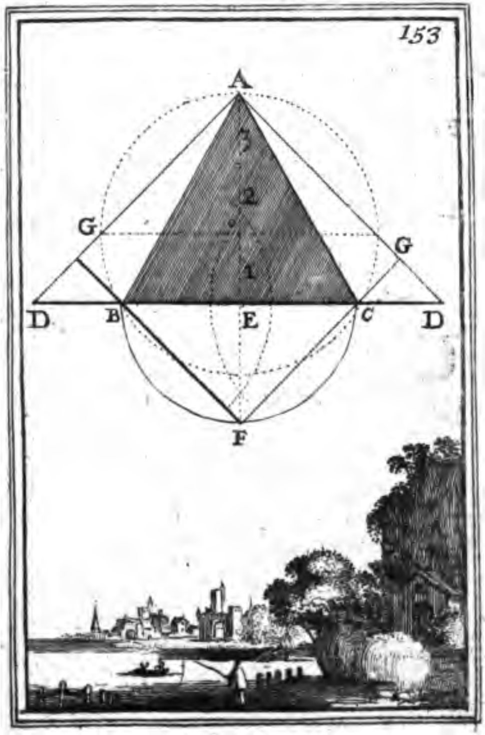
PROPOSITION VII.

To circumſcribe a ſquare about a given equilateral triangle.

Let A B C be the equilateral triangle, about which a ſquare is to be circumſcribed.

OPERATION.

Page 58. BIſect the baſe	B C
in the point	E
Produce the baſe	B C
both ways towards	D & D
Make the lines	E D & E D
equal to the line	E A
Upon the point	E
wtth the diſtance	E C
Deſcribe the ſemicircle	B F C
draw the line	A E F
From the point	F
draw the lines	F C G & F B G
and the ſquare requir'd will be	A G F G


153
A
3
2
G
G
1
D
B
E
C
D
F

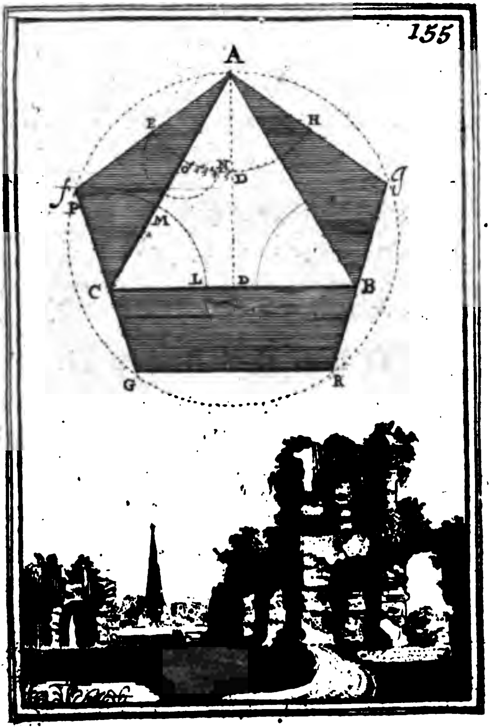
PROPOSITION VIII.

To circumſcribe a pentagon about an equilateral triangle.

Let ABC be the triangle given, about which a pentagon is to be circumſcribed.

OPERATION.

Upon the points or angles	A, B, C
and with the ſame opening of the compaſſes deſcribe at pleaſure the arcs	DE, LP
Divide the arc	D O
into five equal parts	1, 2, 3, 4, 5
upon the center or ſection	O
And with the diſtance of four parts	O N
deſcribe the arc	N M E
Draw the right line	A E f
Cut off the arc	M P
equal to the arc	E N
Draw the right line	f P C g
equal to the line	f A
Make the arc	D H
equal to the arc	D E
Draw the ſides	AI, I R
equal to the ſides	Af, f G
The ſide	I R
will compleat the pentagon demanded.	


155
A
E
H
N
D
f
P
g
M
L
D
C
B
G
R

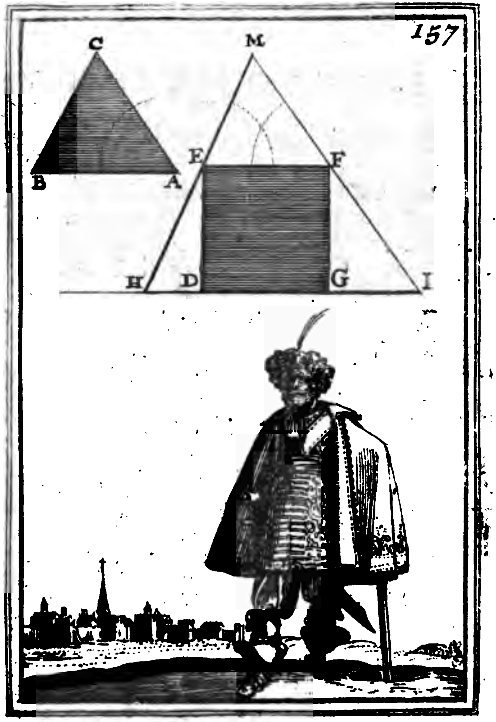
PROPOSITION IX.

To circumſcribe a triangle ſimilar to a given triangle, about a ſquare.

Let D E F G be the ſquare about which a triangle is to be circumſcribed ſimilar to the triangle A B C.

OPERATION.

Page 62.	MAke the angle	E F M
	equal to the angle	A
Page 62.	Make the angle	M E F
	equal to the angle	B
	Produce the lines	M E, M F, D G
	towards	I & H
	H I M will be the triangle requir'd, ſimilar to the triangle	A B C
	and circumſcrib'd about the ſquare	D E F G


157
C
M
E
F
B
A
H
D
G
I

PROPOSITION X.

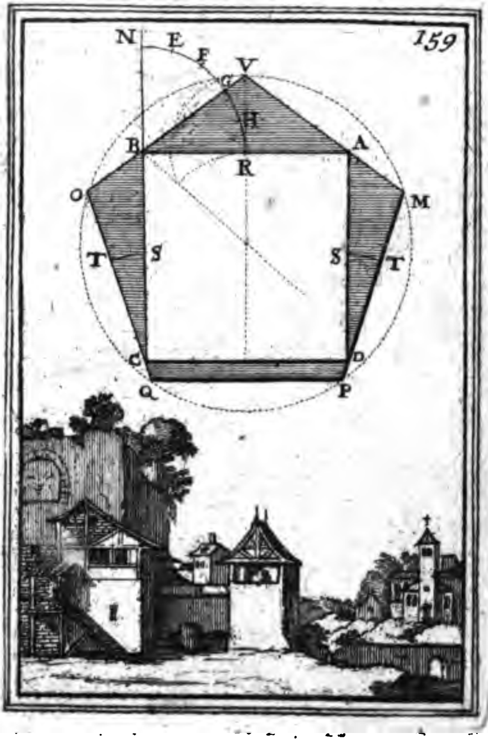
To circumſcribe a pentagon about a ſquare.

Let A B C D be the ſquare about which a pentagon is to be circumſcribed.

OPERATION.

	Produce the ſide	C B
	towards	N
Page 58.	Biſect the ſide	A B
	in the point	R
Page 56.	Erect the perpendicular	R V
	upon the points	B, D, C
	with the diſtance	B R
	Deſcribe the arcs	R N, S T, S T
	Divide the arc	R N
	into five equal parts	R H, G F, E N
	Make the angle	R B V
	with the diſtance of two parts	R G
	Make the angles	S C T, S D T
	with the diſtance of one part	R H
	Produce the lines	V B, C T to O
	Make the line	O Q
	equal to the line	O V

Draw the other ſides after the ſame manner, and you will have the thing requir'd.


159
N
E
F
V
G
H
B
A
R
O
M
T
S
S
T
C
D
Q
P

THE
FIFTH BOOK,
OF
Proportional LINES.

BOOK the FIFTH.

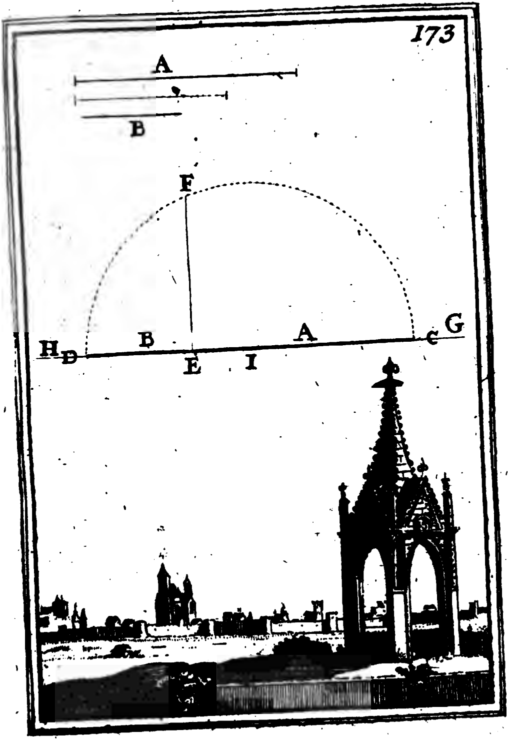
PROPOSITION I.

To find a mean proportional between two given lines.

Let A & B be the two lines between which a mean proportional is to be found.

OPERATION.

Draw an indetermin'd line	G H
Make	C E
equal to the line	A
Make	E D
equal to the line	B
Page 58. Biſect	C D
in the point	I
upon the point	I
and with the diſtance	I C
Deſcribe the ſemicircle	C F D
Erect the perpendicular	E F
This line	E F
ſhall be a mean proportional between	A & B


173
A
B
F
H
D
B
E
I
A
C
G

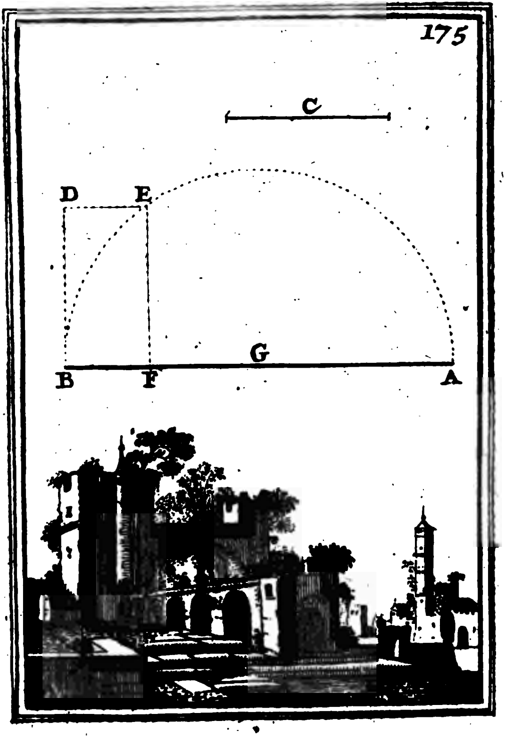
PROPOSITION II.

Given the ſum of the extremes and the mean proportional to diſtinguiſh the means.

Let A B be the ſum of the extremes (*i. e.* the two magnitudes connected without any diſtinction) and C the mean proportional, by whoſe aſſiſtance the point, where the extremes join, is to be diſtinguiſhed.

OPERATION.

Pag. 58.	Biſect the line	A B
	in the point	G
	upon the point	G
	with the interval	G A
	Deſcribe the ſemicircle	A E B
	Erect the perpendicular	B D
	equal to the mean proportional	C
Page 56.	Draw the line	D E
	parallel to the line	A B
	from the ſection	E
Page 56.	Draw the line	E F
	parallel to the line	B D
	Then will the point where the extremes join be	F
	ſo that C or its equal	E F
	ſhall be a mean proportional	
	between	A F & B F


175
C
D
E
G
B
F
A

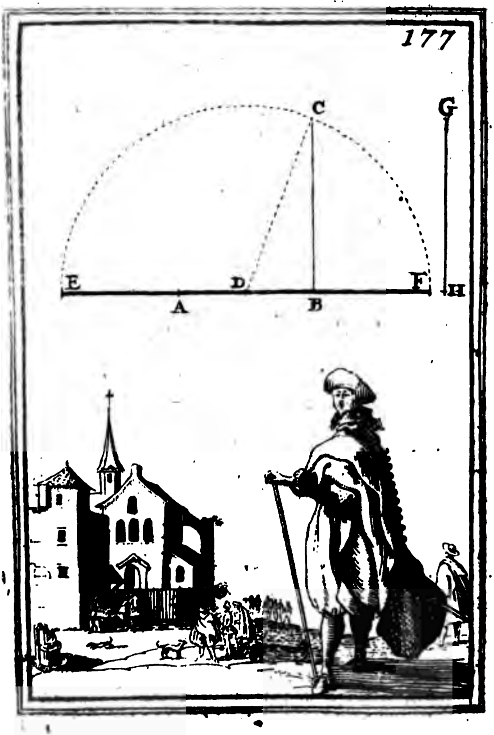
PROPOSITION III.

Given the mean of three proportionals and the difference of the extremes, to find the extremes.

Let G H be the mean proportional, and A B the difference of the extremes, requir'd the length of the extremes.

OPERATION.

Page 50.	ERect the perpendicular	B C
	at the extremity of the difference	A B
Page 58.	and equal to the mean	G H
	Biſect the difference	A B
	in the point	D
	Produce both ways towards	E & F
	upon the point	D
	with the diſtance	D C
	Deſcribe the ſemicircle	E C F
	The extremes requir'd will be	B E, B F


177
C
G
E
D
F
H
A
B

PROPOSITION IV.

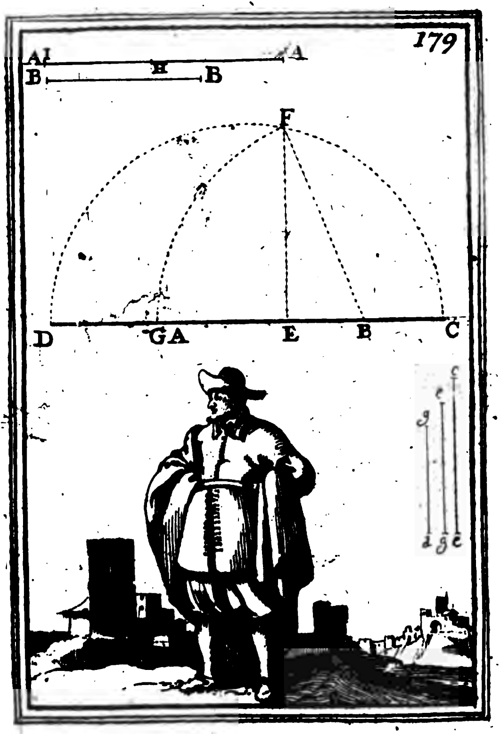
To cut off from a given line, a part that ſhall be a mean proportional between what remains and another given right line.

Let A A be the line, of which a part is to be cut off, that ſhall be a mean proportional between what remains and the line propoſed B B

OPERATION.

DRaw the indefinite line	C D
cut off the lines	D E, E C
equal to the lines	A A & B B
Deſcribe the ſemicircle	C F D
Page 46. Erect the perpendicular	E F
Page 58. Biſect the line	C E
in the point	B
upon the point	B
with the diſtance	B F
Deſcribe the arc	F G
Cut off the part demanded	A H
equal to the part	E G

A H will be the mean proportional between the remainder H I and the other line propoſed B E


179
A I
A
H
B
B
F
D
G A
E
B
C
g
e
c
d
g
e

PROPOSITION V.

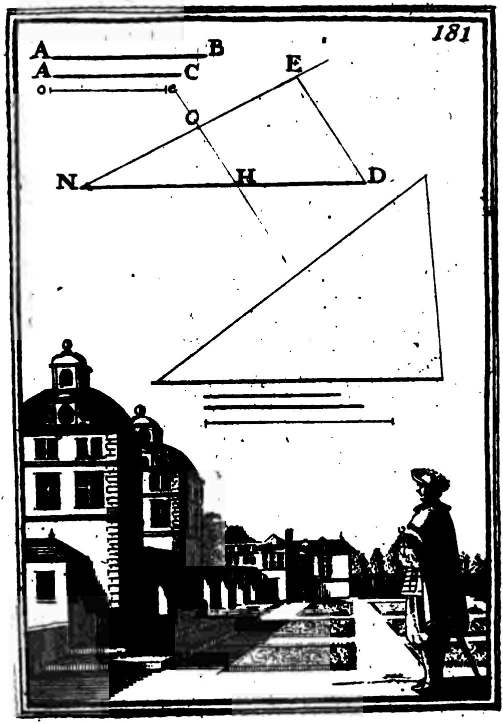
Two right lines being given to find a third proportional.

A B, A C are the two given right lines, to which a third proportional is to be found.

OPERATION.

Make at pleaſure the angle	D N E
Cut off the part	N H
equal to the line	A B
Cut off the part	N O
equal to the line	A C
Draw the line	H O
Page 56. Draw the line	D E
parallel to the line	H O

E O will be the third proportional requir'd.


181
A
B
A
C
E
O
N
H
D

PROPOSITION VI.

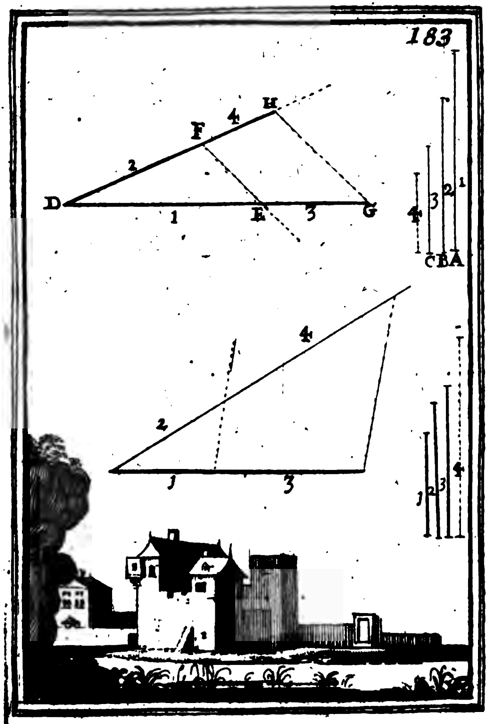
To find a fourth proportional.

A, B, C, are the three lines propoſed, a fourth is to be found, which will be to the third juſt as the ſecond is to the firſt.

OPERATION.

Make at pleaſure the angle	G D H
Cut off the part	D E
equal to the line	A
Cut off the part	D F
equal to the line	B
Cut off the part	E G
equal to the line	C
Draw the line	E F
Page 56. Draw the line	G H
parallel to the line	E F

F H will be the fourth proportional demanded.


H
F
D
E
G
C B A

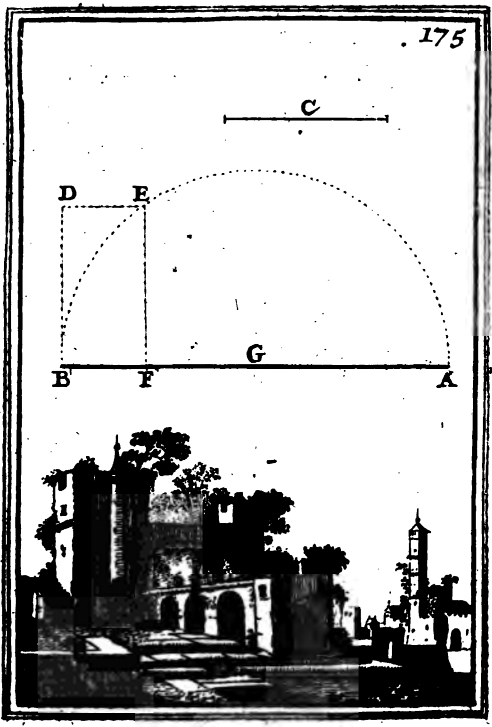
PROPOSITION VII.

To find two mean proportionals between two given lines.

Let I & H be the lines propoſed, between which two mean proportionals are to be found.

OPERATION.

	DRaw the line	A B
	equal to the line	H
	Let fall the perpendicular	B C
	equal to the line	I
	Draw the line	A C
Page 58.	Biſect the line	A C
	in the point	F
Page 50.	Erect the perpendiculars	A O, C R
	upon the point or center	F
	Deſcribe the arc	D E
	ſo that the chord	D E
	may touch the angle	B
	A D, C E will be the mean proportionals between the given lines	I & H


175
C
D
E
G
B
F
A

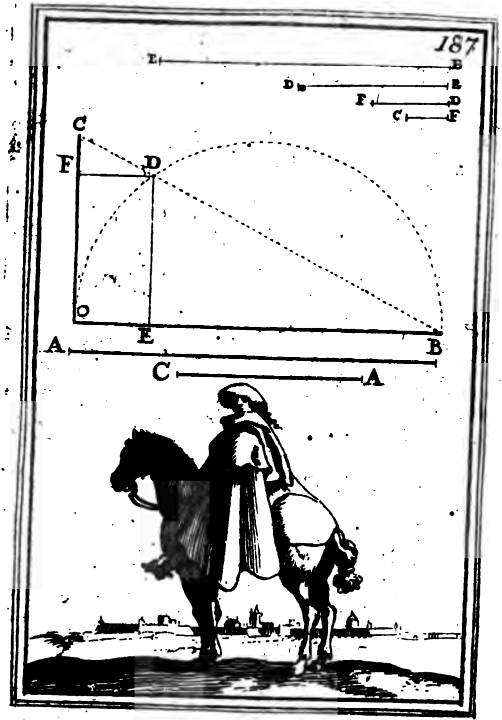
PROPOSITION VIII.

To cut two given lines, each into two parts, ſo as that the four ſegments may be proportional.

A B, A C are the lines propoſed to be cut according to the propoſition.

OPERATION.

	Make the right angle	B O C
	Cut off the line	B O
	equal to the line	A B
	Cut off the line	O C
	equal to the line	A C
	Draw the hypothenuſe	B C
	Deſcribe the ſemicircle	B D O
	from the ſection	D
Page 56.	Draw the line	D E
	parallel to the line	C O
Page 56.	and the line	D F
	parallel to the line	E O
	A B will be cut in	E
	O C alſo in	F
	ſo that B E will be to	E D
	as E D to	D F, & E D
	to D F, as D F is to	F C


187
E
B
D
E
F
D
C
F
C
F
D
O
E
B
A
B
C
A

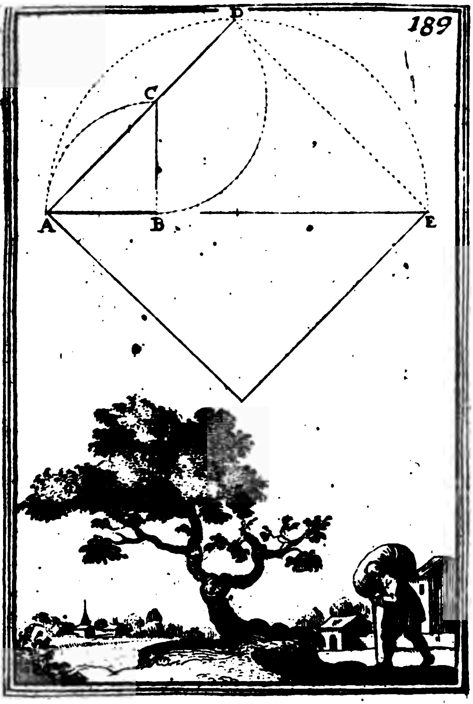
PROPOSITION IX.

The exceſs of the diagonal of a ſquare, above the ſide being given, to find its ſide.

Let A B be the exceſs of the diagonal of a ſquare above its ſide, to find its magnitude.

OPERATION.

Page 50. Erect the perpendicular	B C
equal to the exceſs	A B
Draw the line	A C
produced towards	D
upon the point	C
and with the diſtance	C B
Deſcribe the arc	B D
A D will be the ſide of the ſquare	A
the exceſs	A B
of whoſe diagonal	A E
above the ſaid ſide	A D


189
D
C
A
B
E

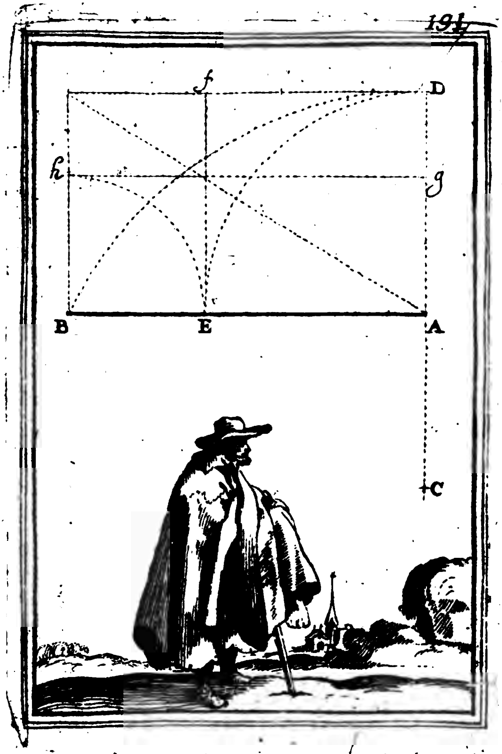
PROPOSITION X.

To cut a given finite line in extreme and mean proportion.

Let A B be the line that is to be cut, ſo that the rectangle of the whole line, and one of the parts may be equal to the ſquare of the other.

OPERATION.

Pag. 50. ERect the perpendicular	A D
Produce it towards	C
Make	A C
equal to half	A B
Upon the point	C
and with the diſtance	C B
Deſcribe the arc	B D
upon the point	A
with the diſtance	A D
Deſcribe the arc	D E
The line	A B
will be cut in the point	E
in the proportion requir'd: for if you make the rectangle A h of the whole A B and part B E, it will be equal to the ſquare A f made upon the other part	A E


191
f
D
h
g
B
E
A
C

PROPOSITION XI.

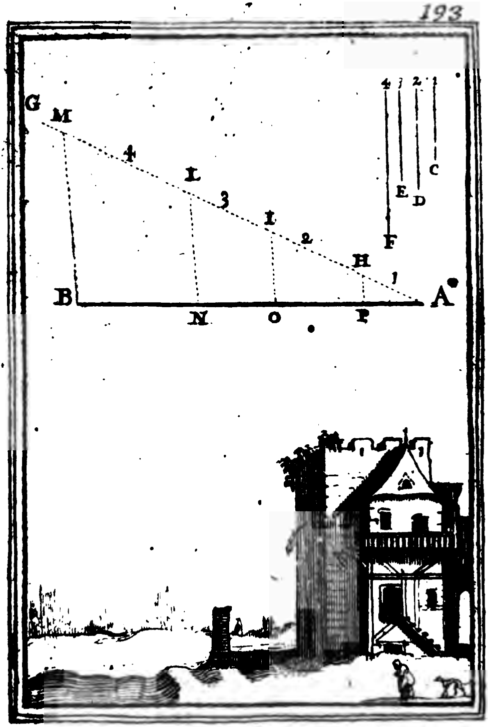
To divide a given right line in any ratio propoſed.

Let A B be the line propoſed to be divided according to the ratios of C, D, E, F.

OPERATION.

Upon the point or extremity	A
Draw at pleaſure the line	A G
Make	A H
equal to the line or ratio	C
Make	H I
equal to the line	D
Make	I L
equal to the line	E
Make	L M
equal to the line	F
Draw the line	B M
Page 50. Draw the lines	L N, I O, H P
parallel to the line	B M

The line AB will be divided in the points P,O,N according to the ratio demanded.



N.4

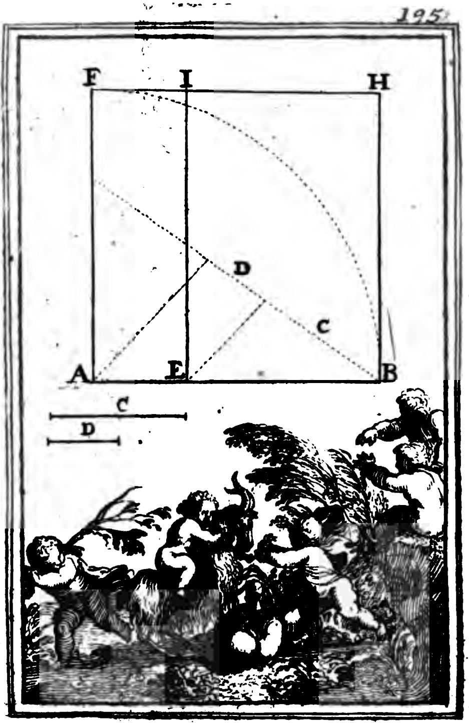
PROPOSITION XII.

To make upon a given right line two rectangles, that ſhall be in any given ratio to one another.

Let A B be the line upon which two rectangles are to be made, which ſhall be to one another as C to D

OPERATION.

Page 184	DIvide the right line	A B
	at the point	E
	in the ratio of	C to D
Page 82.	Make the ſquare	A B H F
Page 56.	Draw the line	E I
	parallel to the line	A F
	B E I H, A E I F will be the rectangles requir'd.	
	the rectangle	A I
	is to the rectangle	E H
	As the line	D
	is to the line	C


195
F
I
H
D
C
A
E
B
C
D

THE

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